PROCEEDINGS OF THE 3rd INTERNATIONAL CONFERENCE OF THE INSTITUTE OF OPERATIONAL RESEARCH AND MANAGEMENT SCIENCE OF NIGERIA (IORMS)

Hosted Virtually From The Headquarters

UNIVERSITY OF LAGOS, AKOKA, LAGOS, NIGERIA

between

17TH to 19TH November, 2020

Theme:

Overcoming impacts of COVID-19: an Operational Research optimisation approach

Institute of Operational Research and Management Science

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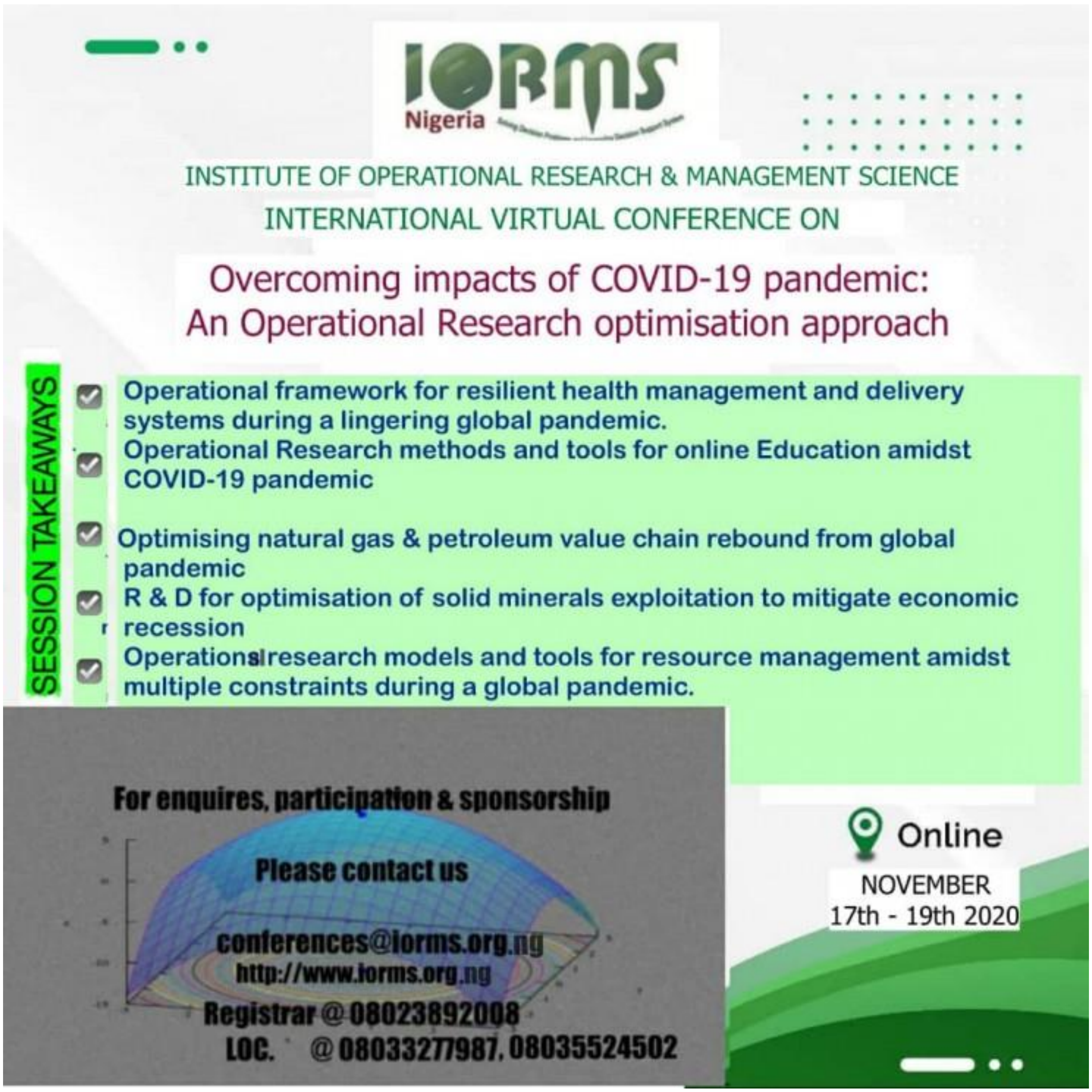

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IORMS Nigeria

e-Conference 2020

HOME TO THE DECISION SUPPORT SYSTEM

PROGRAMME OF EVENTS

DAY 1 TUESDAY 17TH NOVEMBER 2020

CAT	OPENING SESSION
12:00 - 12:05	Conference Opening - 2nd stanza of National Anthem / Anchor welcome participant
12:05 - 12:15	Chairman 2020 conference committee- Gbenga Oso, FNCS Declaration of Conference Open- NOC Chairman Prof. R.K. Ojikutu
12:15 - 12:25	Chief Host Remarks-Vc University of Lagos-Prof. Toyin Ogundipe 10mins
12:25 - 12:45	Goodwill messages / acknowledgements- Registrar IORMS- Okesola Moses, CMC (5mins) Goodwill Address: DVC DS (Fmr Ag. VC) UNILAG - Prof. Folasade Ogunsola (15 mins)
12:45 – 13:05	Welcome Address : President IORMS - Prof Rasheed Ojikutu (20mins)
	PLENARY SESSION 1
13:00 - 13 :05	Chairman: Prof Rasheed Ojikutu
13:05 - 13:30	Guest Speaker: Prof. Adedeji B. Badiru Operational Research methods & tools for online Education amidst COVID-19 pandemic (25mins) AFIT Air Force Institute of Technology OHIO, USA
13:30-13:45	Comments & Responses (15mins)
13:45 -14:00	Conferment of Fellowship Award/ Day One Closing

DAY 2 WEDNESDAY 18TH NOVEMBER 2020

12:00:12:15	Registration and acceptance/Awareness/Welcoming guest - Anchor: Dr. S. Adewoye	
12:15 - 12:30	COMMENCEMENT OF PARALLEL SESIONS SPECIAL ANNOUNCEMENTS & INSTRUCTIONS ON GROUPING/ DISTRIBUTION Anchor/Compere: Dr. S. Adewoye, Sec. General, IORMS	
12:30- 12:35	ROOM A1 Chairman: Prof. Rasheed Ojikutu Moderator: Dr. O. S. Asaolu	ROOM B1 Chairman: Dr. F. O. Ogunwolu Moderator: Mr. Okesola Moses
12:35 - 12:50	Paper 1 Predictive Modelling of COVID-19 pandemic evolution in Nigeria E. Agunloye & M. Usman University of Lagos	Paper 2 Application of queueing models for evaluation of effectivenessof the automatic disinfectant booths in Federal Polytechnic Ilaro O. Ogunsanwo & H. Akinwale Federal Polytechnic Ilaro
12:50 - 13:00	Q/A	Q/A

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	DAY 2 WEDNESDAY 18TH NOVEMBER 2020 CONTINUATION	
13:00-13:05	ROOM A1 Chairman Dr. I. Etukudo Moderator: Dr. O. S. Asaolu	ROOM B1 Chairman Dr. M. Adamu Moderator: Mr. Okesola Moses
13:05-13:20	Paper 3 **Resilient Health Systems to Mitigate Impact of Pandemic on Medical Professionals** Dr. Omotola O. Oso Nigeria Army Reference Hospital e e ee , 68NARH Yaba	Paper 4 **COVID-19: Imperative for E-Service Quality Improvement in Business Organizations** Ighomereho O. Salome PhD Redeemer's University, Ede, Osun State, Nigeria
13:20-13:30	Q/A	Q/A
13:30-14:00	Break out session/ Operations Research Awareness display	
	PLENARY SESSION 2	
14:00-14:05	Chairman/Moderator: DR. O. S. Asaolu, 1st Vice President IORMS	
14:05-14:30	Guest Speaker: Prof. Christopher Thron 25mins Optimal management of COVID-19 using a combination of distancing & testing strategy	
14:30-14:45	Comments & Responses. (15mins)	
	DAY 2 CLOSING	

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PROGRAMME OF EVENTS

DAY 3 THURSDAY 19TH NOVEMBER 2020

12:00-12:15	Registration and acceptance/Awareness/Welcoming guest - Anchor: Dr. S. Adewoye	
	PLENARY SESSION 3	
12:15-12:20	Chairperson/Moderator: Dr. B. Dixon-Ogbechi, B. N (Mrs.), (Asso. Professor,UNILAG)	
12:20-12:45	Guest Speaker: Dr.Paula Carroll (Asst. Professor) Chair Euro Women in Society Doing OrMs (WISDOM) Sustainability modelling and optimisation contribution to affordable and clean energy (25mins) University College Dublin Ireland's Global University	
12:45-13:00	e e ee Comments & Responses (15mins)	
13:00-13:10	BREAK-OUT FOR PARALLEL SESIONS 2/OR AWARENESS	
13:10-13:15	ROOM A2 Chairman: Dr. S. Adewoye Moderator: Dr. Dixon-Ogbechi, B. N. (Mrs.)	ROOM B2 Moderator/Chairman: Mr. Okesola Moses Moderator: Mr. Marshal Sampson
13:15-13:30	Paper 5 A Comparative Analysis of the shortest Path Algorithm for COVID-19 activities & distribution of palliatives items Dr. Abraham Tamber, Ikpotokin Festus, Okafor Linus, Odeh Jane. Benue State University Ambrose Allí University Nigeria Defence Academy	Paper 6 Operations Research Models and Tools for Resource Management amidst multiple constraints in Organisations . Calyxtus Onyeunoegbunem Enweazu
13:30-13:40	Q/A	Q/A

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PROGRAMME OF EVENTS

DAY 3: THURSDAY, NOVEMBER _19_2020_CONTD.	
13:40-14:00	**BREAK-OUT FOR NEXT SESSION /OR AWARENESS**
14:00-14:05	**ROOM A2 (combined)** **Chairman:** **Mr. Okesola Moses** **Moderator: Mr. Marshal Sampson**
14:05-14:20	**Paper 7** **Enumerative numerical solution for optimal control using treatment and vaccination for an SIS epidemic models** **Prof. Christopher Thron** **Texas A & M University** **Central Texas, USA** e e ee
14:20-14:30	**Questions and Answer session**
	INDUCTION OF NEW MEMBERS
14:30-14:50	**-Welcome Remark:** President IORMS - Professor Rasheed Ojikutu **- Professional Ethics & Discipline Remarks:** **- Presentation of Candidates for Induction Pledge** **- Issuance of Membership Certificate** Registrar IORMS- Okesola Moses, CMC **Response by representative of Inductees** **Closing/Vote of Thanks** **Mr. Felix Amadi 2nd Vice President IORMS** **NATIONAL ANTHEM**
14:50-15:30	**ANNUAL GENERAL MEETING**

PROGRAMME OF EVENTS

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2020 Conference Inductees

Fellowship Category

Oso Olugbenga Ayoola	F2020001

FULL Member Category

Ighomedreho O. Salome(Ph.D)	M2020001
Ofuani, Barbara Aiwanehi(Ph.D)	M2020002
Adebola, Glorious Adekoya	M2020003
Abraham Jighjigh .Tamber Victor	M2020004
Effiong Ekpo	M2020005.
Kingsley Mordi	M2020006.
Okoh Godwin Luke	M2020007
Adewusi Adewale	M2020008
Edu Semiu Olatunde	M2020009

Associate Member Category

Ogbechi Daniel Chinaedum Ayo.	A2020001

Student Member Category

Ogbechi Valentino Olisemeka O.	S2020001
Ogbechi Naomi Olulu Boluwatife	S2020002

IORMS Officials

National Organising Committee Members

Professor R. K. Ojikutu	**President**
Dr. O. S Asaolu	**1st Vice President**
Chief Felix Amadi	**2nd Vice President**
Dr. Dixon-Ogbechi B. N. (Mrs.)	**Treasurer**
Dr. S. Adewoye	**Secretary**
Mr. M. Okesola	**Registrar**
Mr. S.Marshal	**Pro**
Dr. I. Etukudo	**South-South Coordinator**
Mr. I. Shehu	**North West Coordinator**
Mr. Gbenga Oso	**South West Coordinator**

Local Organising Committee Members

Mr. Gbenga Oso	**Chairman**
Dr. O. S Asaolu	**Member**
Dr. Dixon-Ogbechi B. N.(Mrs.)	**Member**
Mr. M. Okesola	**Member**
Mr. S.Marshal	**Member**

END OF CONFERENCE 2020

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IORMS 2020 Conference Schedule

DAY 1

IORMS Conference is inviting you to a scheduled Zoom meeting.

Topic: IORMS 2020 Virtual Conference Opening Day 1

Time: Nov 17, 2020 12:00 PM Africa/Lagos

Join Zoom Meeting

https://us02web.zoom.us/j/7982166519?pwd=WVpGSThCQWNkUDljME5xVHppQ0tYdz09

Meeting ID: 798 216 6519

Passcode: bzN025

DAY 2

MAIN EVENT

Topic: IORMS 2020 Virtual Conference Day 2 Main

Time: Nov 18, 2020 12:00 PM Africa/Lagos

Join Zoom Meeting

https://us02web.zoom.us/j/7982166519?pwd=WVpGSThCQWNkUDljME5xVHppQ0tYdz09

Meeting ID: 798 216 6519

Passcode: bzN025

PARALLEL SESSIONS

ROOM A1

Topic: IORMS 2020 Virtual Conference Day 2 Room A1

Time: Nov 18, 2020 12:00 PM Africa/Lagos

Join Zoom Meeting

https://us02web.zoom.us/j/85890136942?pwd=T1JOK2d2c1lPRGx5K092N1NUa2tyUT09

Meeting ID: 858 9013 6942

Passcode: 929854

ROOM B1

Topic: IORMS 2020 Virtual Conference Day 2 Room B1

Time: Nov 18, 2020 12:00 PM Africa/Lagos

Join Zoom Meeting

https://us02web.zoom.us/j/87299386031?pwd=VUdYbTFLUmMxQXc2bm4yN05WUG14dz09

Meeting ID: 872 9938 6031

Passcode: 391355

DAY 3

MAIN EVENT

Topic: IORMS 2020 Virtual Conference Day 3 Closing

Time: Nov 19, 2020 12:00 PM Africa/Lagos

Join Zoom Meeting

https://us02web.zoom.us/j/7982166519?pwd=WVpGSThCQWNkUDljME5xVHppQ0tYdz09

Meeting ID: 798 216 6519

Passcode: bzN025

PARALLEL SESIONS

ROOM A2

Topic: IORMS 2020 Virtual Conference Day 3 Room A2

Time: Nov 19, 2020 12:00 PM Africa/Lagos

Join Zoom Meeting

https://us02web.zoom.us/j/87596289555?pwd=cldhRzRHMHErRG9ZTHBBcExBL0JHdz09

Meeting ID: 875 9628 9555

Passcode: 235574

ROOM B2

Topic: IORMS 2020 Virtual Conference Day 3 Room B2

Time: Nov 19, 2020 12:00 PM Africa/Lagos

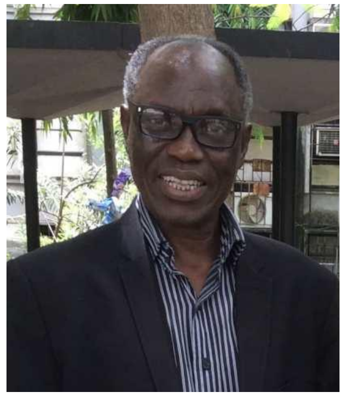
Join Zoom Meeting

https://us02web.zoom.us/j/85982217375?pwd=eHZYMEd0NmY5YlBheWNGam54M3hJQT09

Meeting ID: 859 8221 7375

Passcode: 168838

President's Welcome Address



The Governor of Lagos State, Mr. Olusola Babajide Sanwolu,

The Chairman of Governor's Forum and Governor of Ekiti State, Dr Kayode Fayemi ,

Our Host, the Vice Chancellor of the University of Lagos, Prof Oluwatoyin Ogundipe,

The Vice Chairman of IORMS, Dr Asaolu,

Distinguished Members of IORMS,

Members of the Press,

Ladies and Gentlemen.

I welcome you all to the third International Conference of the Institute of Operational Research and Management Sciences of Nigeria. The role of Operational Research in resource management cannot be over-emphasized . Therefore, experts in this unique area of human endeavor and professional uniqueness are expected to raise the intellectual bar by rising to the occasion in defense of the people of the world whenever random shocks attack the commonwealth , scarce resources and endowments bestowed on humanity. It is in this regard that the theme of this year's conference titled "OVERCOMING the impact of COVID 19 pandemic: An Operational Research optimization approach" is contemporary, timely and most appropriate.

Humanity has faced serious and dangerous challenges since the emergence of Coronavirus in Wuhan, China in February 2019. The epidemic that emerged from that remote corner of the world has transformed into a global pandemic with immense effect and with such rapidity that has left humanity with its mouth agape. The fact that the research world with its huge intellectual endowment and human capital have not been able to emerge from the rubbles of the devastation and wanton damage unleashed on it by the virus with a positive cure almost two years after its discovery shows the level of unpreparedness and confusion to which the human community had been subjected during the period. Unfortunately, the virus continues to rides freely over the global terrain damaging without serious check the human and material resources of the world.

For months, the global productions line of the world were locked down, factories, offices , schools transportation and human movements and mobilities and all engines that make humanity thrive were completely shut down . Consequently, the world economy nose-dived to near ground zero with such rapidity that palpitates the human heart.

The World Health Organization statistics shows that as at 14thNovember 2020, over 53,174,803 confirmed cases of Coronavirus were recorded all over the world with 1,300,576 deaths. There were 657,312 New Cases. The Americas is leading the pack with 22,707,430 confirmed cases, followed by Europe with 14,792,945, South-East Asia has 9,964,225, the East Mediterranean with 3,512,233, Africa has 1,393,792 and West Pacific with 793,437 confirmed cases. The statistics of mortality in these regions are equally damning while the world is expected to face enormous reduction in its Gross Domestic Product (GDP) which resultantly will have dare and lasting consequences on the well- being of the people.

In all these and with the emergence of fresh cases, it doesn't look as if public health measures put in place to stem the tide have succeeded in completely eradicating the scourge of the pandemic. As at today, the fear of renewal and resurgence of the virus with renew vigor all over the world is becoming a reality. The question is "With the economy of the world already in a prostrate position and resources at the near bottom of its container, can the world survive another global lockdown?"

With near total collapse in human output and a consumption pattern that is not dwindling, one can foresee a disaster to the human race if decisive management sciences tools are not quickly employed to match production with consumption. Operational Research techniques are available to provide answers to several questions in this area.

The three-day Conference of IORMS which comes up from November 17 to November 19, 2020 is designed to provide answers to some of the socio- economic challenges posed by scourge of COVID 19 .The lead paper on the first day , November 17, 2020 titled " ***Operational Research Methods and tools for online education amidst COVID 19 pandemic***" will be delivered by Professor Adedeji .Badiru of the Air Force Institute of Technology, Ohio, USA. This will be followed by four parallel sessions on day 2 in the morning and a plenary session in the afternoon with a paper titled " ***Optimal Management of COVID 19 using a combination of distancing and testing strategy"*** delivered by Professor Christopher Thron of the Texas A&M University, Central Texas, USA.

On November 19,2020, that is on day 3, Dr Paula Carroll, an Associate Professor at the University College Dublin, Ireland and the chair of WISDOM (Women in Society Doing OrMs) will deliver the paper titled " ***Sustainability Modeling and Optimization Contribution to***

affordable and Clean Energy" at the third plenary session. This will also be followed by two parallel sessions.

I do hope that all of you present at this opening ceremony will stay glued to the Programme at it unfolds it's academic and intellectual contents to this challenging societal and global problems.

I cannot conclude this address without paying glowing tribute to Late Professor Olufunsho Akingbade, the founding father of IORMS whose doggedness and resilience make it possible for us to gather for knowledge sharing on this occasion. May his soul Rest in Peace.

Once more I welcome you to the third international annual conference of the Institute of Operational Research and Management Sciences of Nigeria. I wish you a happy and fruitful deliberation.

Professor Abdur Rasheed Kola Ojikutu

President IORMS.

LOC Chairman's Remarks

All Protocols observed

It gives me optimal pleasure to welcome you all to this unique, first of its kind IORMS international conference. It is our 3rd in the series but the 1st virtual due to the unprecedented and ravaging covid-19 pandemic.

We just listened to the opening prayer by the 2nd stanza of the National Anthem urging God of creation, to direct our noble cause, and likewise, he has given us the duty to bring a positive change to the World. One of such changes is to apply Operational Research tools like optimisation, forecasting, and modeling to affect optimal management decisions.

So, our theme this year is "**Overcoming the impact of covid-19 pandemic: An operational research optimisation approach**".

We thank you all for finding time to attend this conference and I urge you all to stay with us throughout to collaborate, educate, and be inspired.

I would like to thank the President of IORMS, NOC, and particularly the LOC members for making this conference a reality. Let me now invite our ever supportive and able President, Professor R.K. Ojikutu to declare this 2020 virtual Conference Open.

Thank you all.

Olugbenga Ayoola Oso (FNCS, MNIM)

Chairman Local Organising Committee

Operational Research Methods & Tools for Online Education Amidst COVID-19 Pandemic

Adedeji B. Badiru
Air Force Institute of Technology

IORMS (Nigeria) Virtual Conference
Lagos, Nigeria
November 17-19, 2020

INTRODUCTION

It is a systems world these days due to the increasing worldwide connectivity that is facilitated by fast travel modes and Internet connectivity. As a result, education requires a new paradigm. In as much as the curriculum is the weapon of accomplishing education, the methodology of this session centers on a systems framework for applying operational research methods and tools for online education amidst COVID-19 pandemic. Curricula of nowadays must take advantage of the emerging transition to digital engineering, artificial intelligence techniques, remote learning, modeling, simulation, and optimization techniques. We cannot afford for education to be static. Existing and new curricula must be upgraded to be adaptive, resilient, and responsive to developments around the world, at a fast pace. The emergence of COVID-19 makes it even more imperative that we act fast and responsibly. The specific framework that is presented is the DEJI Systems Model®, a trademarked tool for achieving a structured design, evaluation, justification, and integration of the elements of systems. Any curriculum that is not designed to align with local needs cannot be fully integrated into the socio-economic needs of the local population in any region of the world. This session offers specific actionable and results-based strategies for online education in response to the COVID-19 pandemic.

COVID-19 Pandemic has upended the long-held notions of how we deliver Education.

- We must adapt and be creative in sustaining the education enterprise.
- As we are resorting to online education amidst the pandemic, we must use operational research methods and tools to achieve efficient and effective delivery of the education mission.
- We must use a systems approach to get the most out of our limited resources.

Motivation for Educational Process Improvement

- Resource limitation
- Urgency
- Emergency
- Global demand
- **COVID-19**
- Etc.

"A rule of thumb is that a lousy process will consume ten times as many hours as the work itself requires." - Bill Gates

Foundation for Superior Online Teaching

- Cultural sensitivity
- Appreciation of diversity
- Sustainable operation
- Recognition of socio-economic settings
- Student-friendly pedagogy
- Responsiveness to local needs
- Efficient and Effective operation
- Continuously-improving operation
- Compliance with accreditation standards

Best strategy: **A systems approach** ✔

Methods and Tools the I Recommend:

1. **Systems approach**
2. **Modeling for operational research optimization**
3. **Data analytics for pandemic-centric online education**
4. **Process improvement methodologies**

1. Systems Approach

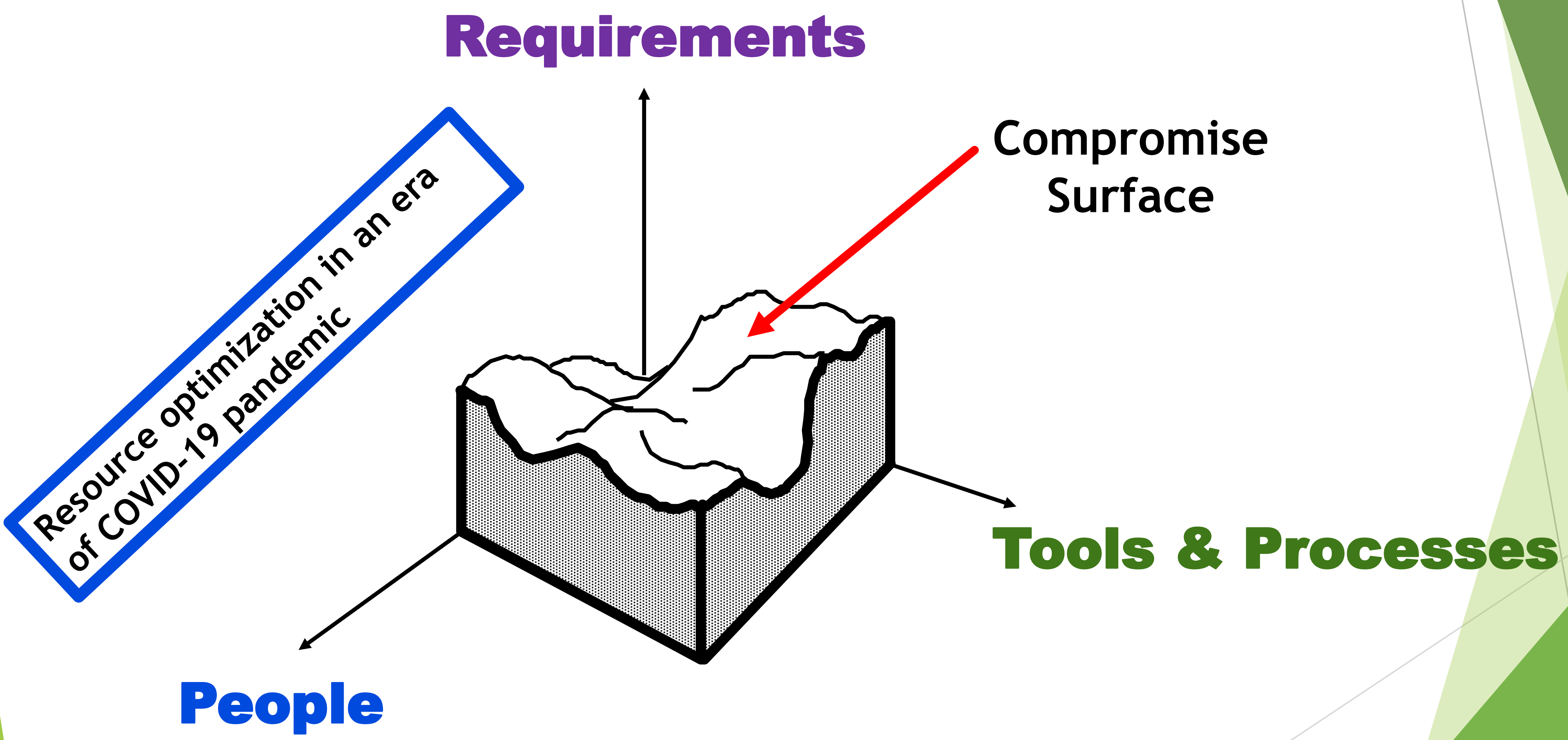
"A system is a collection of interrelated elements, whose collective output, *together*, is higher than the sum of the individual outputs of the elements."

Desirable characteristics of a System:

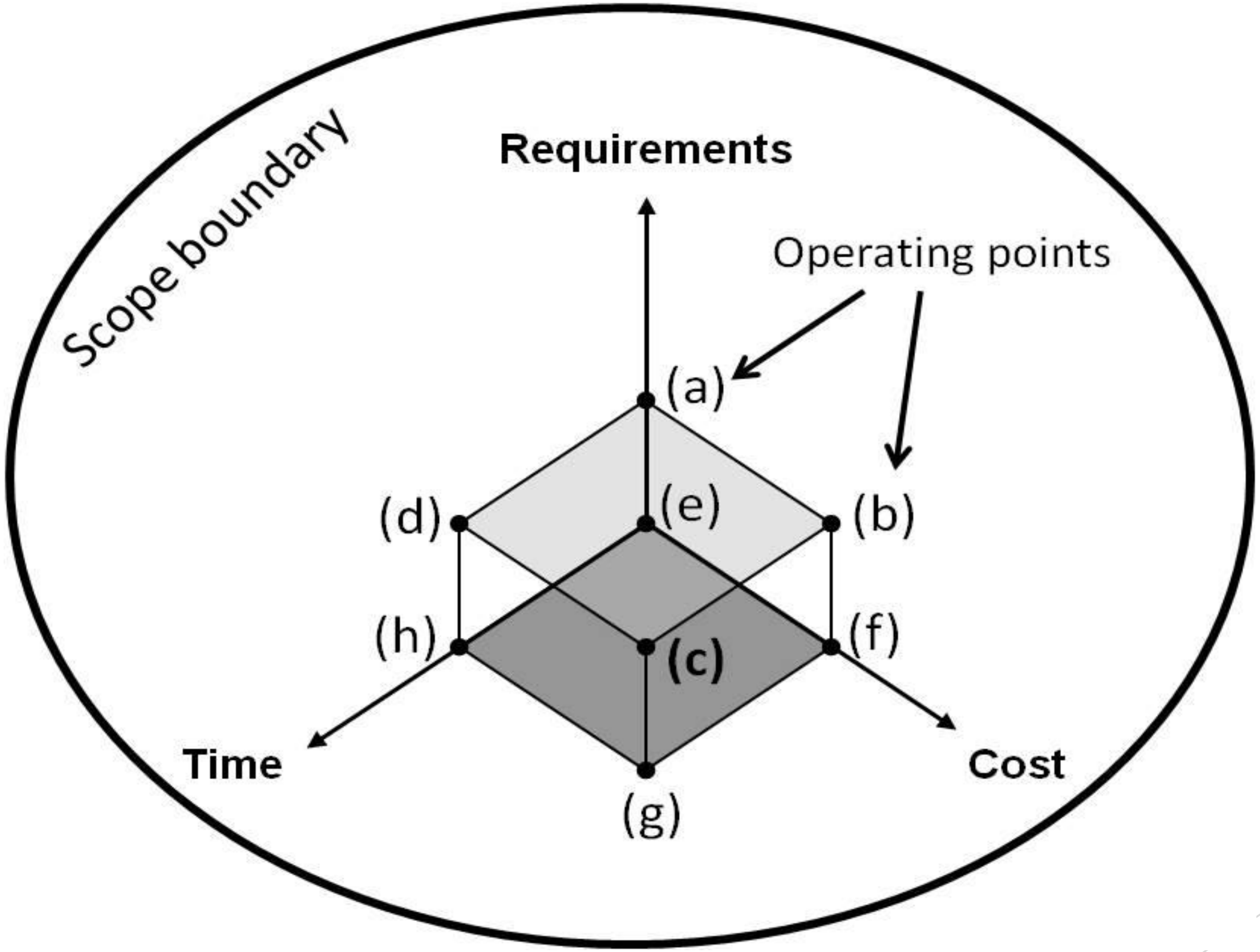
- Self-monitoring
- Self-regulating
- Self-correcting
- Self-advancing
- Self-actuating

Online Education is a typical example of a large, complex, and diverse system

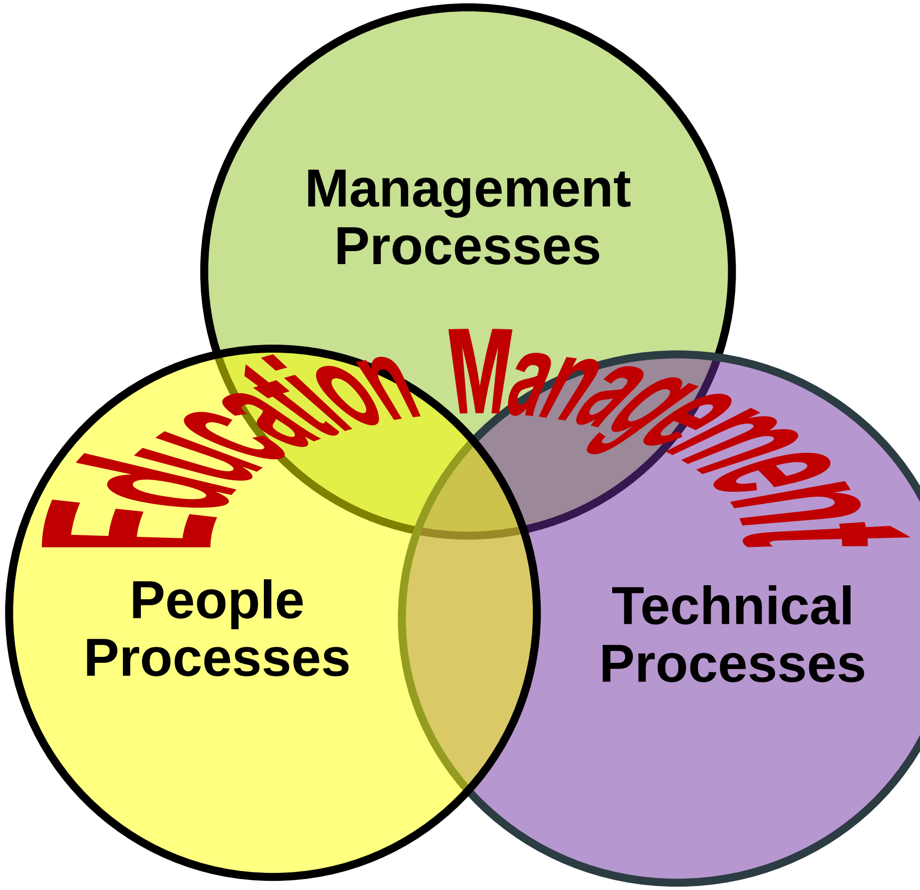
Compromised Profile for Online Education



Optimization Subject to Systems Constraints

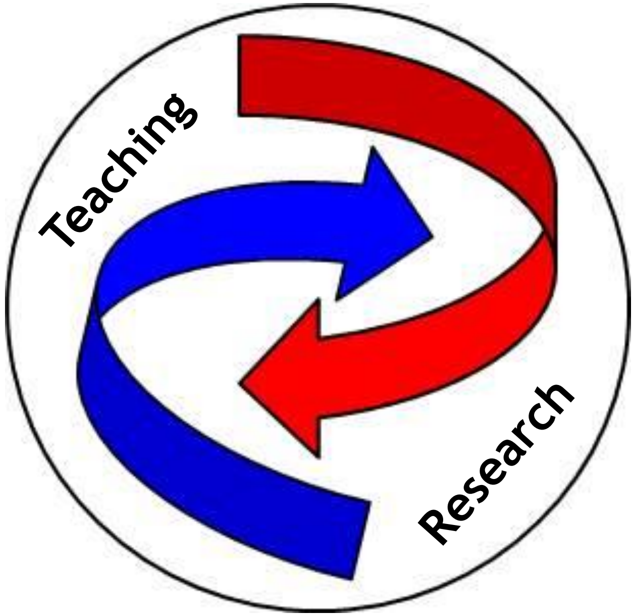


Online Systems Interfaces



People →Tools →Processes

Integration of Research and Teaching: ORMS Imperative

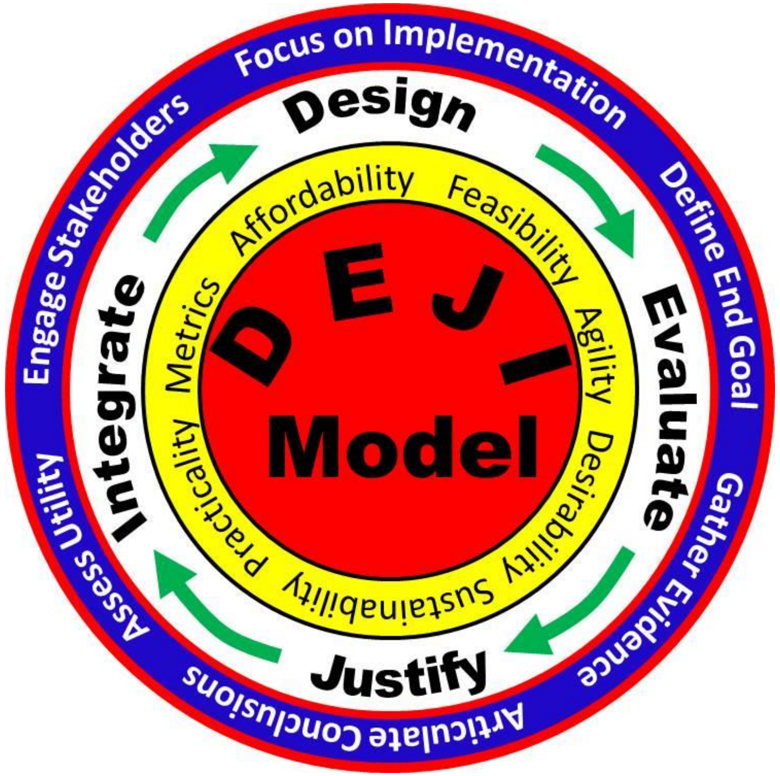


"Research what you teach and Teach what you research."

A Systems Tool for Online Education

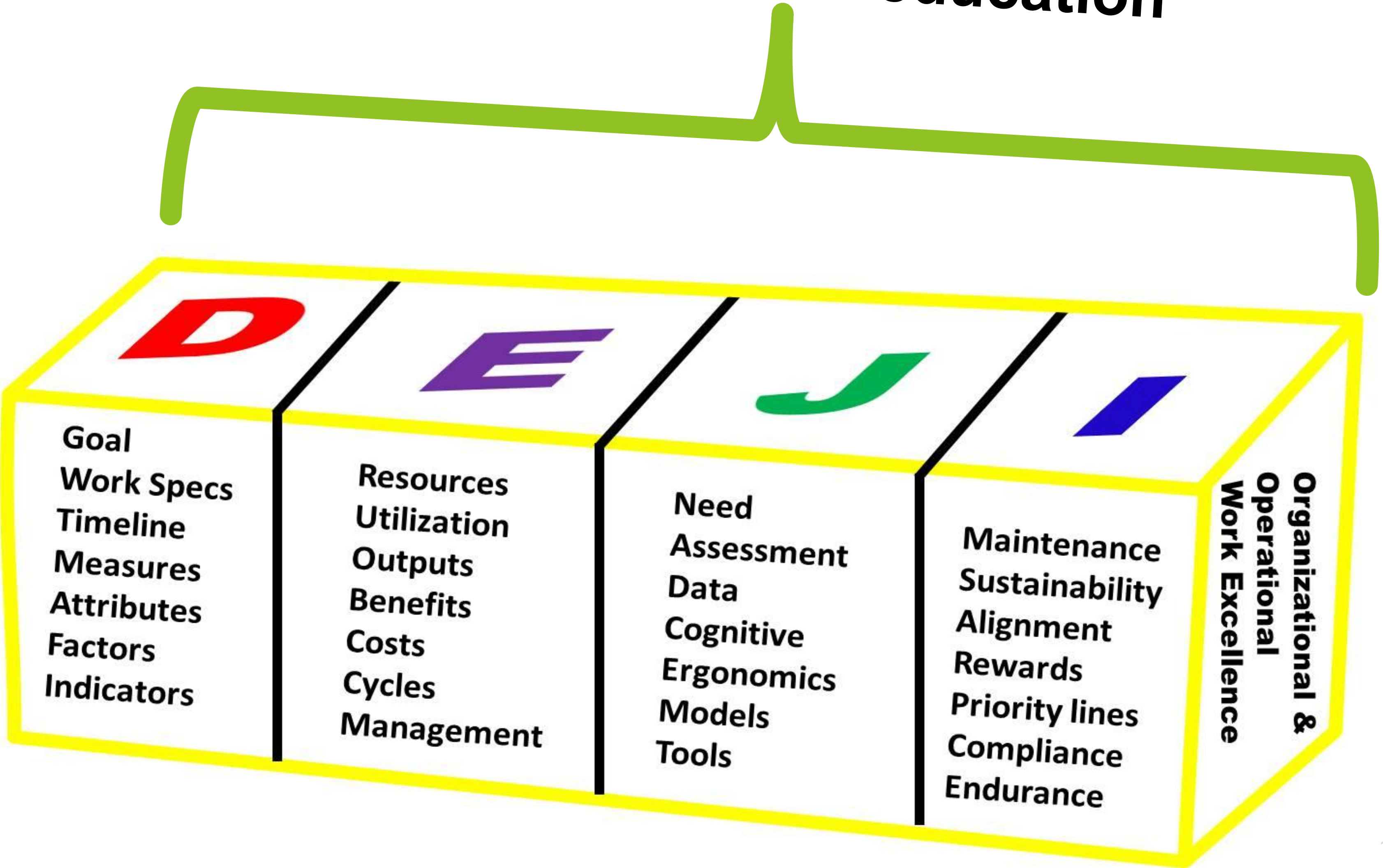
The DEJI® Systems Model

Design → Evaluation → Justification → Integration



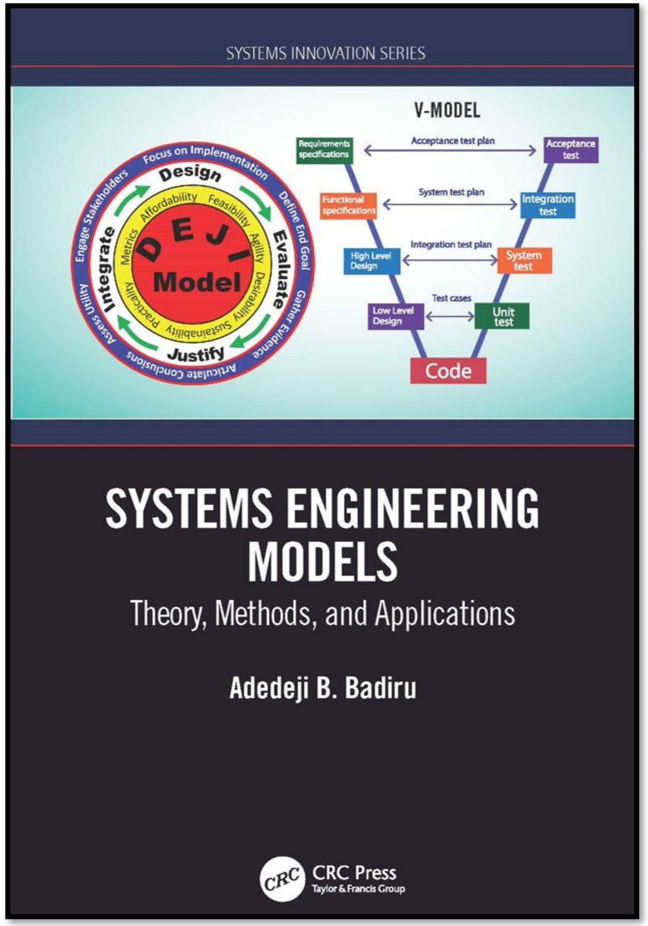
www.DEJImodel.com

Various elements involved in the pursuit of online education



References on DEJI Systems Model:

1. Badiru, A. B. (2014), "Quality Insights: The DEJI Model for Quality Design, Evaluation, Justification, and Integration," ***International Journal of Quality Engineering and Technology***, Vol. 4, No. 4, pp. 369-378.
2. Badiru, Adedeji B. (2019), **Systems Engineering Models: Theory, Methods, and Applications,** Taylor & Francis/CRC Press, Boca Raton, FL.



DEJI® Systems Model Alignment with CDIO ® Framework

DEJI Systems Model® is a trademarked tool for achieving a structured design, evaluation, justification, and integration of the elements of any system.

CDIO (Conceive, Design, Implement, Operate) is a trademarked educational framework for curricular planning that stresses engineering fundamentals in the context of conceiving, designing, implementing and operating real-world systems and products. It originated from MIT in the late 1990s. It is operated as an international collaboration of member universities around the world. It uses active learning tools, such as group projects and problem-based learning, to better equip engineering students with technical knowledge as well as communication and professional skills. It also provides resources for instructors of member universities to improve teaching proficiency.

2. Modeling for Optimization

- Value is a function of Attributes
- Attribute is a function of Factors
- Factor is a function Indicators

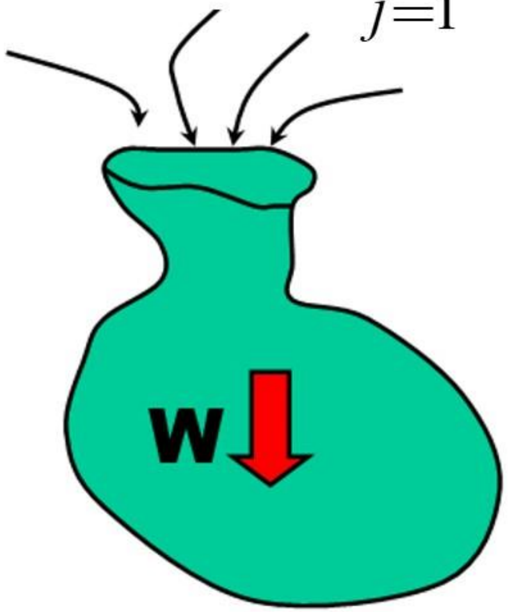
$$V = f\left(A_1, A_2, \ldots, A_p\right)$$

$$A_k\left(x_1, x_2, \ldots, x_{m_k}\right) = \sum_{i=1}^{m_k} f_i\left(x_i\right)$$

$$x_i\left(v_1, v_2, \ldots, v_n\right) = \sum_{j=1}^{n} z_i\left(v_i\right)$$

Knapsack optimization model for total educational value (weight)

$$\sum_{j=1}^{n} c_j x_j$$



Total Educational Value, V

$$V = f\left(A_1, A_2, \ldots, A_p\right)$$

$$= f\left\{\left[\sum_{i=1}^{m_1} f_i\left(\sum_{j=1}^{n} z_j\left(v_j\right)\right)\right]_1, \left[\sum_{i=1}^{m_2} f_i\left(\sum_{j=1}^{n} z_j\left(v_j\right)\right)\right]_2, \ldots, \left[\sum_{i=1}^{m_k} f_i\left(\sum_{j=1}^{n} z_j\left(v_j\right)\right)\right]_p\right\}$$

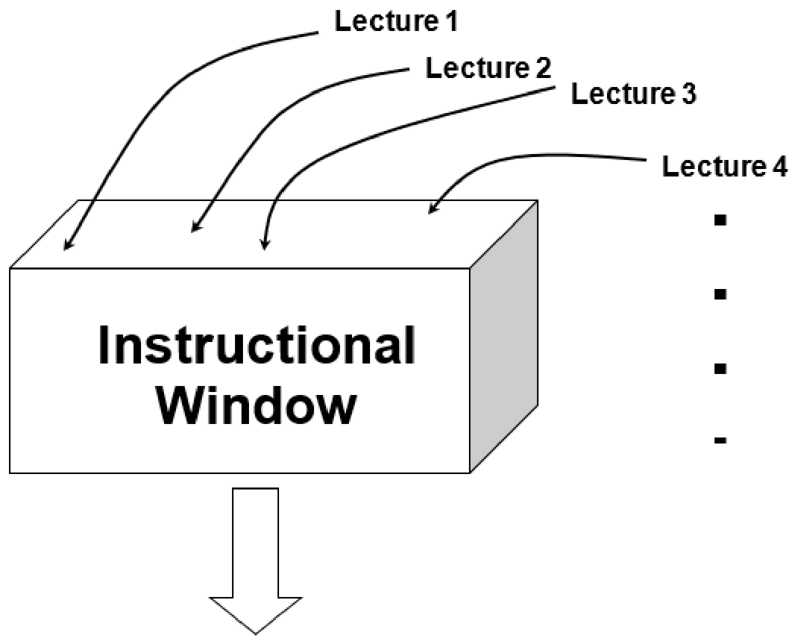
<u>Value Vector</u>

- Attributes (of Value)
- Factors (of Value)
- Indicators (of Value)

<u>Note:</u>

$f_i \longrightarrow f_{(k)i}$, k=1, 2, . . . p

Knapsack Formulation for Classroom Session Windows



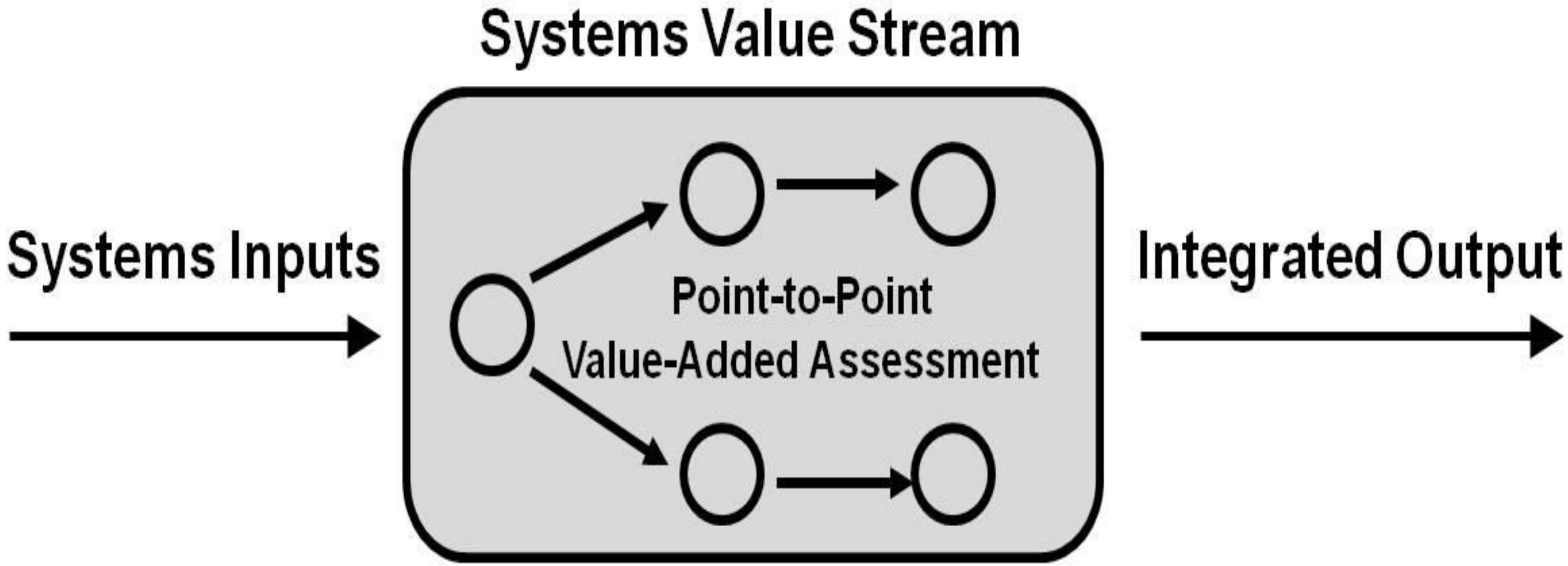
Resource Availability Limitation (W) vis-à-vis learning requirements

Value = priority measure (c_1)
Weight = resource requirement (w_1)
Objective: Maximize value of knapsack without exceeding total weight limit

$$\text{Max} \quad Z = \sum_{j=1}^{n} c_j x_j$$

$$\text{Subject to} \quad \sum_{j=1}^{n} w_j x_j \leq W$$

3. Data Analytics for Online Education Value Stream

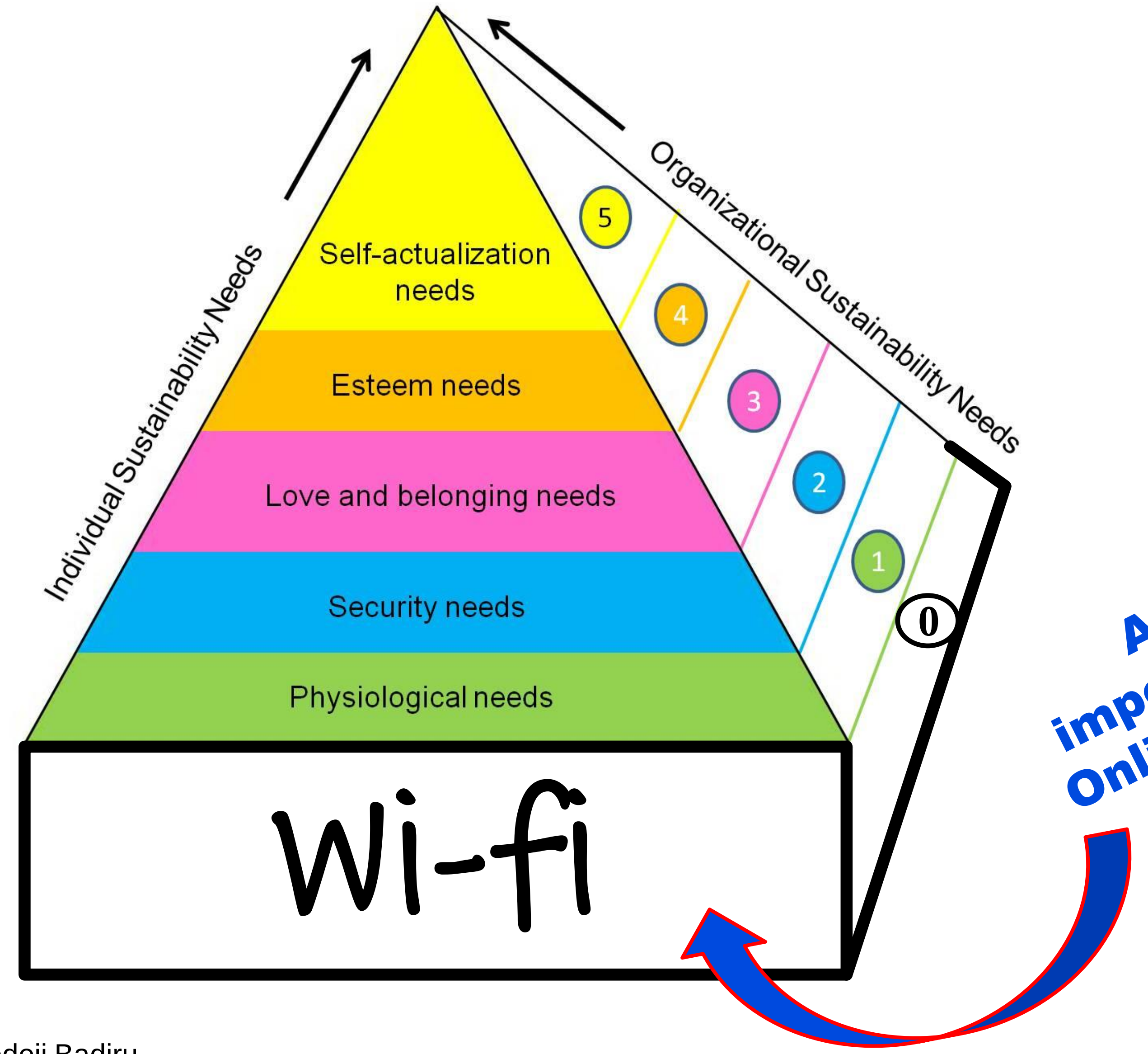


Data on metrics, standards, expectations, and performance

4. Process Improvement Tools

- **PDCA** (Plan, Do, Check, Act)
- **OODA** (Observe–Orient–Decide–Act) loop
- **Lean**
- **Six Sigma**
- **DRIVE** (Define, Review, Identify, Verify, Execute)
- **5S** (Shine, Straighten, Shine, Standardize, Sustain)
- **DMAIC** (Define, Measure, Analyze, Improve, Control)
- **SIPOC** (Suppliers, Inputs, Processes, Outputs, Customers)
- **FMEA** (Failure Mode & Effects Analysis)
- **Fishbone** Diagram
- **Kaizen** (Japanese word for change (kai) for the better (zen))

The *expanded* Hierarchy of Needs for online education



An additional imperative for GEE & Online Learning in the era of COVID-19

Conclusions

→ Questions

→ Discussions

→ Curtains!!!!

Optimal management of COVID-19 using a combination of distancing & testing strategy

IORMS VIRTUAL CONFERENCE 2020

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Vianney Mbazumutima
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Porto Novo, Benin

Much of this material is taken from the extended presentation at:

https://talkcoast.com/ds-i-africa/

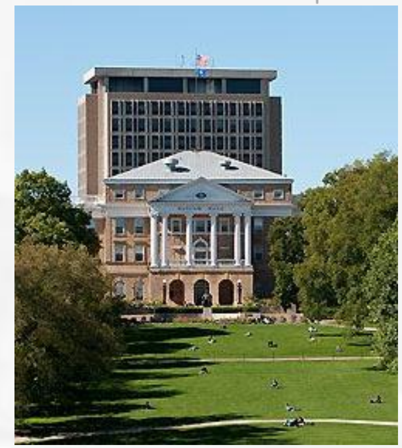
At this site, you may obtain:

- Extended lecture (5 videos, 1 hour total watch time)
- Complete powerpoint,
- Excel model and actual COVID data from NYC and Texas

About me

TEXAS A&M UNIVERSITY CENTRAL TEXAS

Princeton University, B.A. Mathematics (1980)



U. Wisconsin Ph.D. Mathematics (1985)

China (1985-90)

Motorola (1997-2009)

Sudan (2017-2020)

Nigeria (2015-2020)

What are the Limitations of Mathematical Modeling?

Epidemiological mathematical models have limited accuracy

- Inaccurate parameters (known unknowns)
- Model error
  - Approximations
  - Unknown unknowns

Mathematical models should not be taken too seriously.

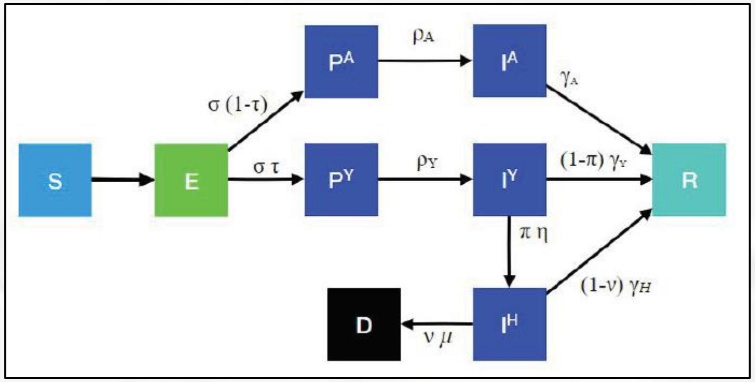
They *cannot* give exact predictions

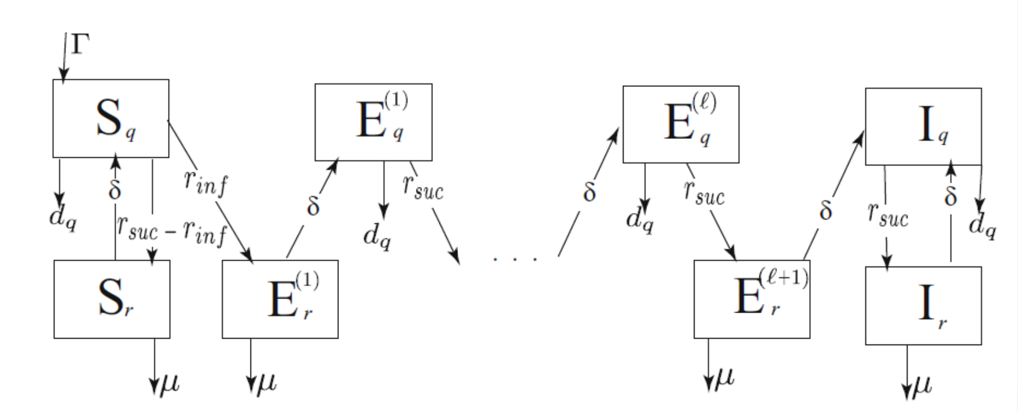
However, they *can* be used for:

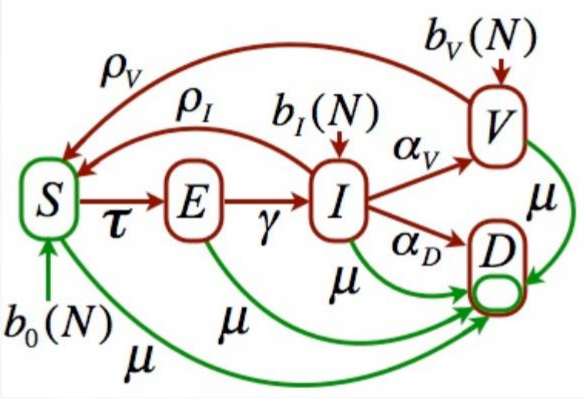
- worst-case scenarios
- sensitivity analysis (the affect of changing different parameters)
- qualitative comparison of treatments' effectiveness and cost

Multicompartment Models

What do these diagrams mean?



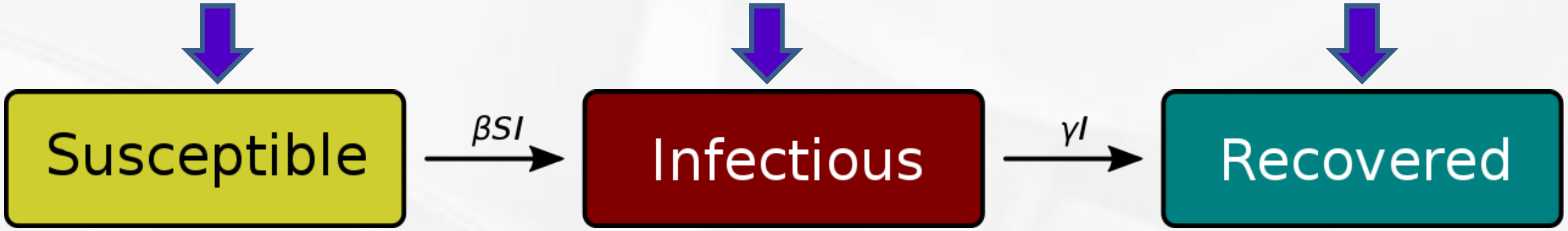




http://dimacs.rutgers.edu/archive/Workshops/ASIEconEpi/Slides/Modeling_Infectious_Diseases.pdf

Basic SIR Model

(Kermack & McKendrick, 1927)



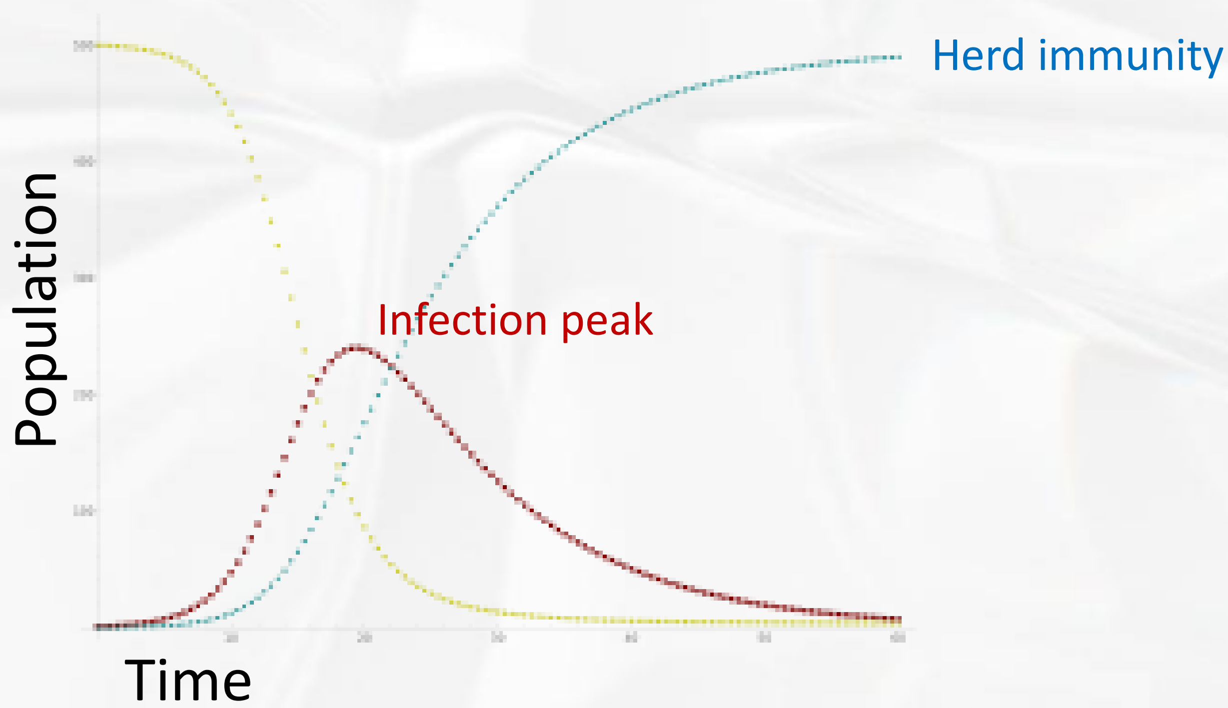
Population is divided into **compartments**

- **Susceptible**: People who may catch the disease
- **Infectious**: People who may infect others
- **Recovered***: People who have recovered from the disease

** In the simplest model, recovered are immune to reinfection*

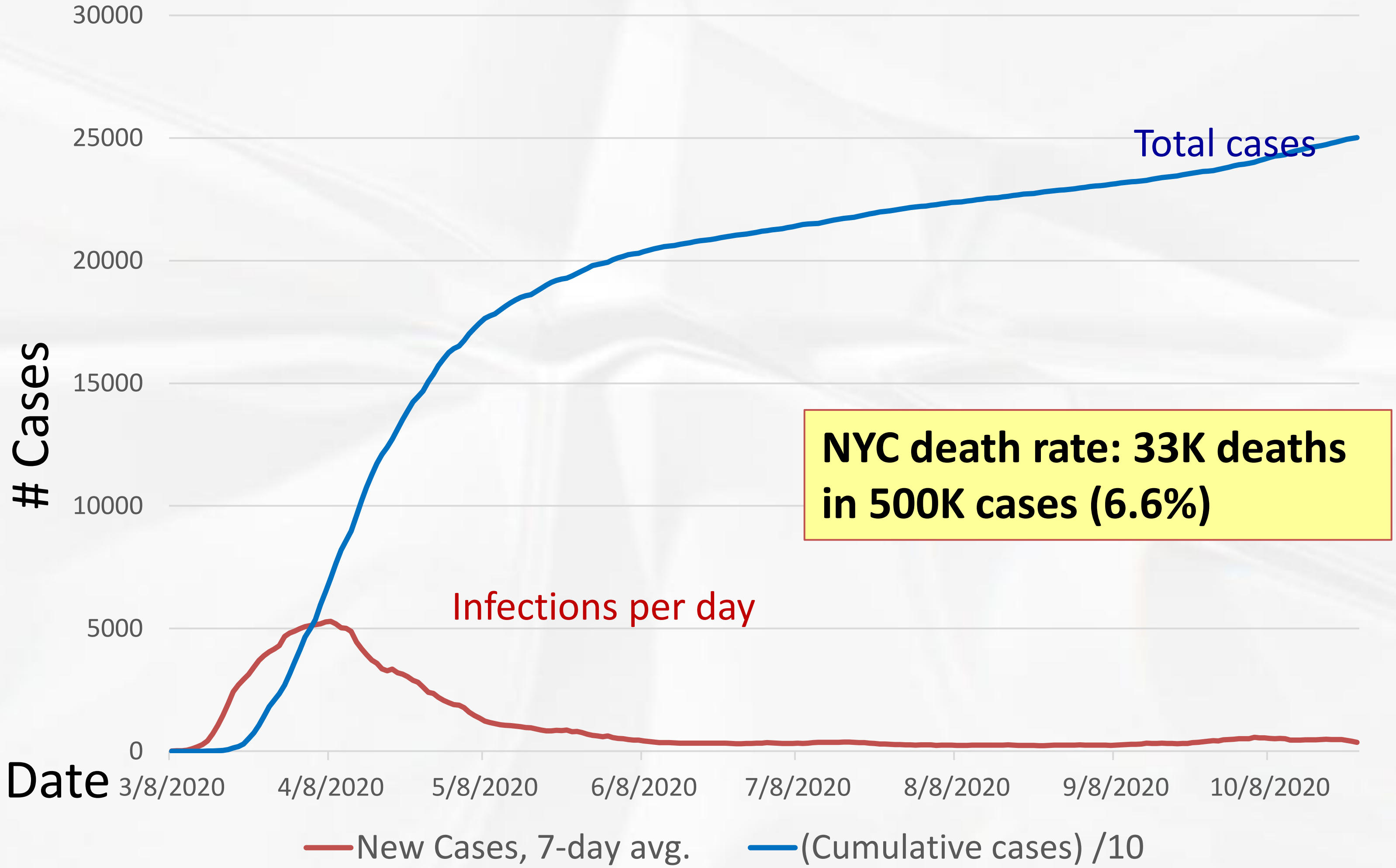
The SIR model describes how the populations in these compartments change over time

Time evolution of SIR Model

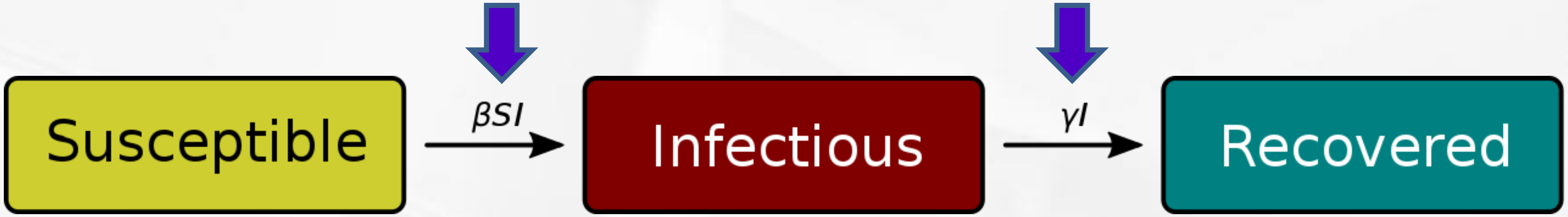


https://en.wikipedia.org/wiki/Compartmental_models_in_epidemiology#The_SIR_model

COVID-19 Cases, New York City, March-October 2020

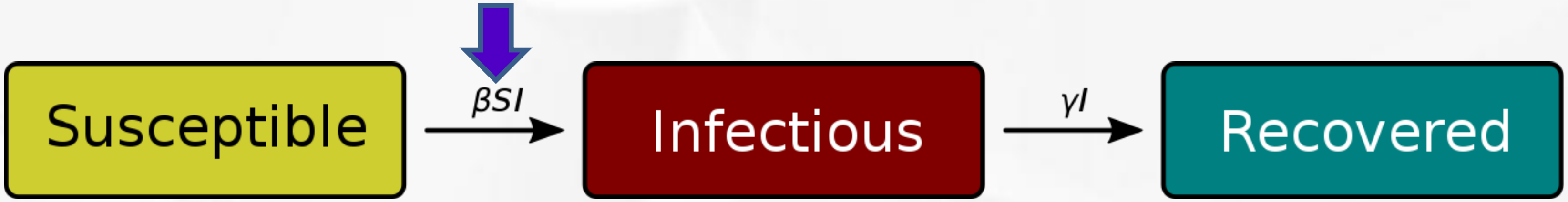


Transition rates



- Individuals move or transition between compartments over time
- Transition rates tell the number of individuals per day that move between compartments
- Rates are proportional to population sizes
- βI (beta$\times$ I) is called the **force of infection**
- γ (gamma) is called the **recovery rate**

How to calculate force of infection



- Each susceptible person contacts a certain number of people per day (*contact rate C)*
- A certain fraction of these contacts are infectious (*infectious proportion equal to I/N)*
- A certain fraction of contacts with infectious people produce disease *(transmission rate T)*

Number of new infections per day

$= S \times C \times (I/N) \times T$

$= (CT/N) \times SI$

$$\beta = \frac{\text{contact rate} \times \text{transmission rate}}{N}$$

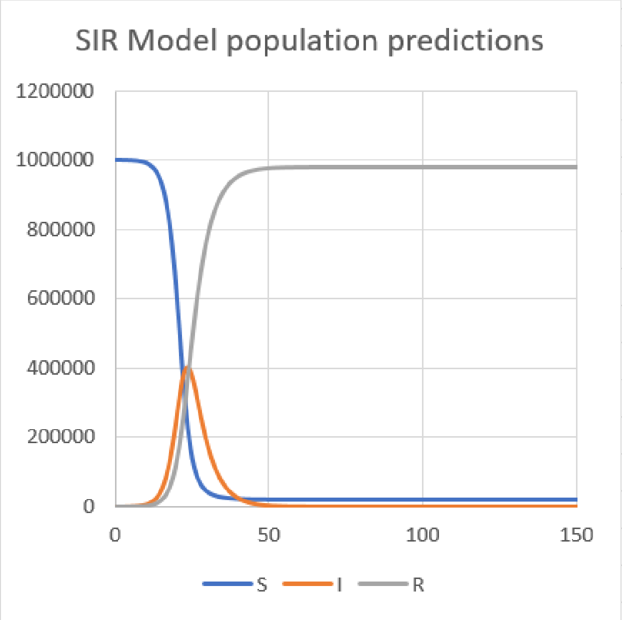
How to calculate recovery rate

- The period of infection is usually *d* days
- So each time period, about 1/*d* infectious people recover

$$\gamma = 1/(\text{average period of infection})$$

Constructing a simple model in Excel

	A	B	C	D	E	F	G
1	**Parameters**			Day #	**S**	**I**	**R**
2	***Population parameters:***			0	999900	100	0
3	Total population	1000000		1	999828	151.99	20
4	*Starting percentages:*			2	999719	231.01	50.399
5	Susceptible %	99.99		3	999552	351.09	96.601
6	Infected %	0.01		4	999300	533.54	166.82
7	Recovered %	0		5	998916	810.71	273.53
8	***Model parameters***			6	998333	1231.7	435.67
9	*Infection parameters*			7	997447	1870.6	682
10	Contacts per day	12		8	996104	2839.9	1056.1
11	Transmission rate	0.06		9	994067	4308.7	1624.1
12	beta	0.00000072		10	990983	6530.8	2485.9
13	*Recovery parameters*			11	986323	9884.5	3792
14	Mean infectious duration	5		12	979304	14927	5768.9
15	recovery rate (gamma)	0.2		13	968779	22467	8754.3
16	*Reversion parameters*			14	953108	33644	13248
17	Mean immunity duration	1E+11		15	930020	50004	19977
18	Reversion rate (rho)	0		16	896537	73486	29977
19				17	849101	106225	44675

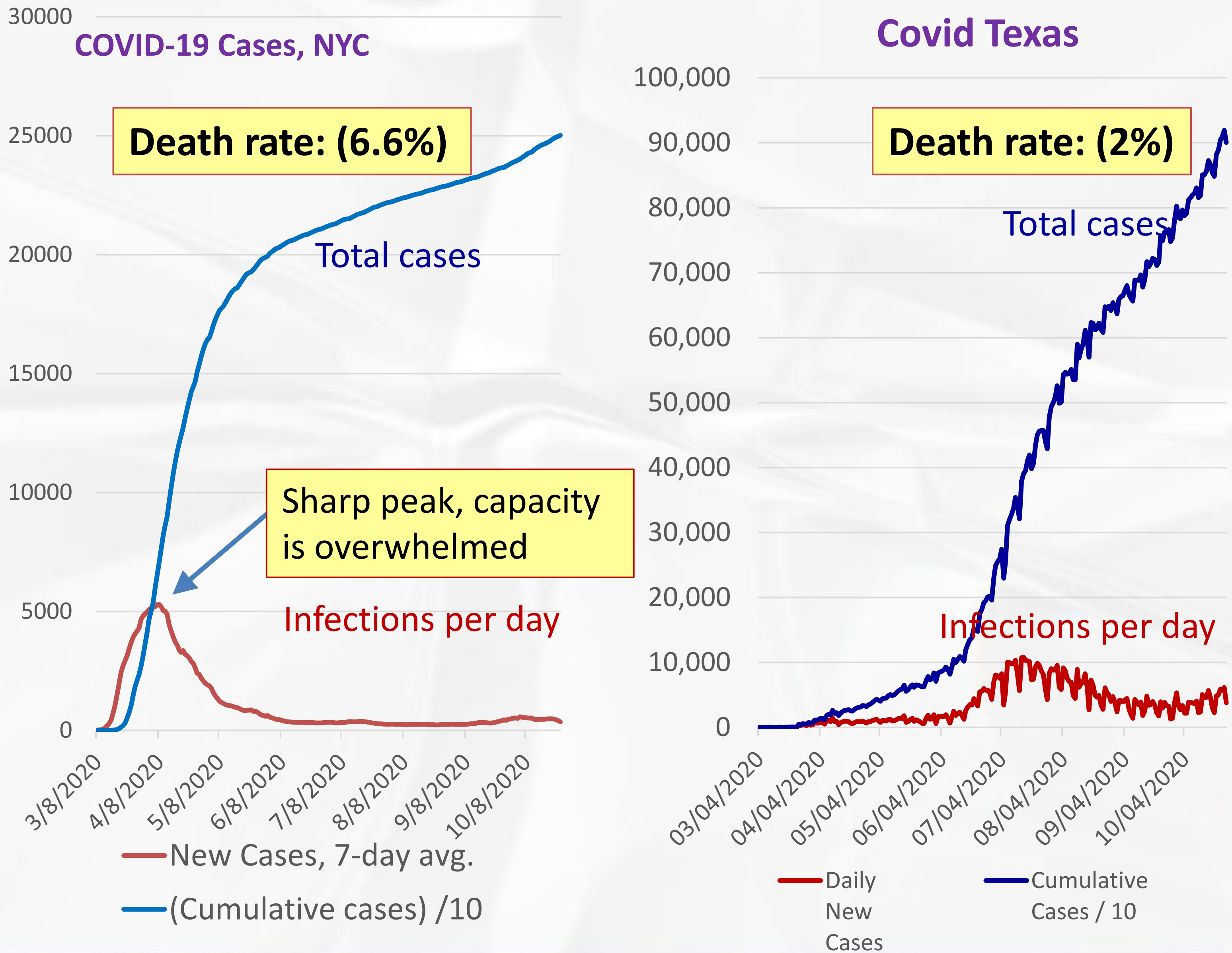


Obtain resources from:

https://talkcoast.com/ds-i-africa/

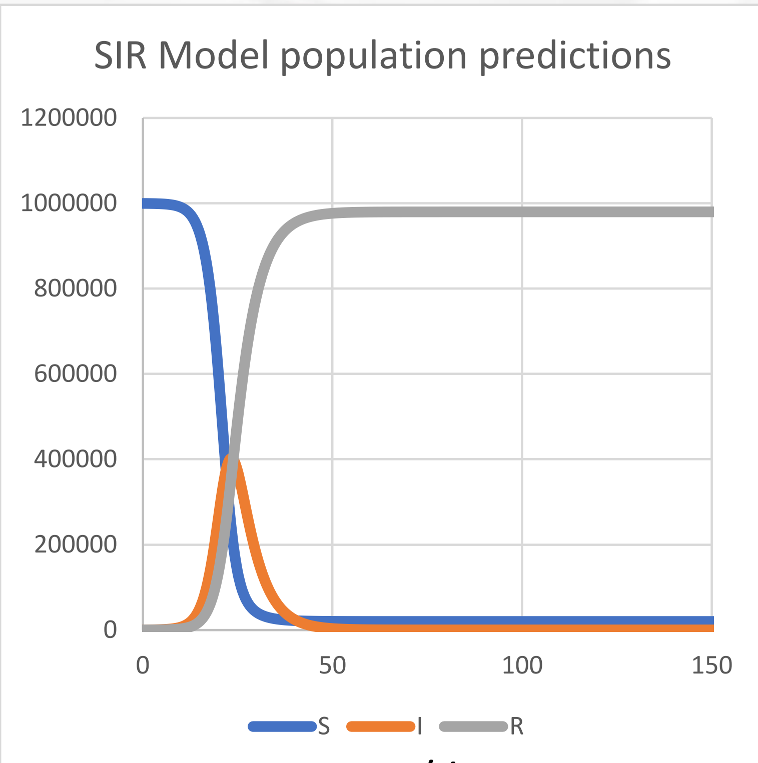
Illustrating epidemiological concepts with the SIR model

Flattening the curve



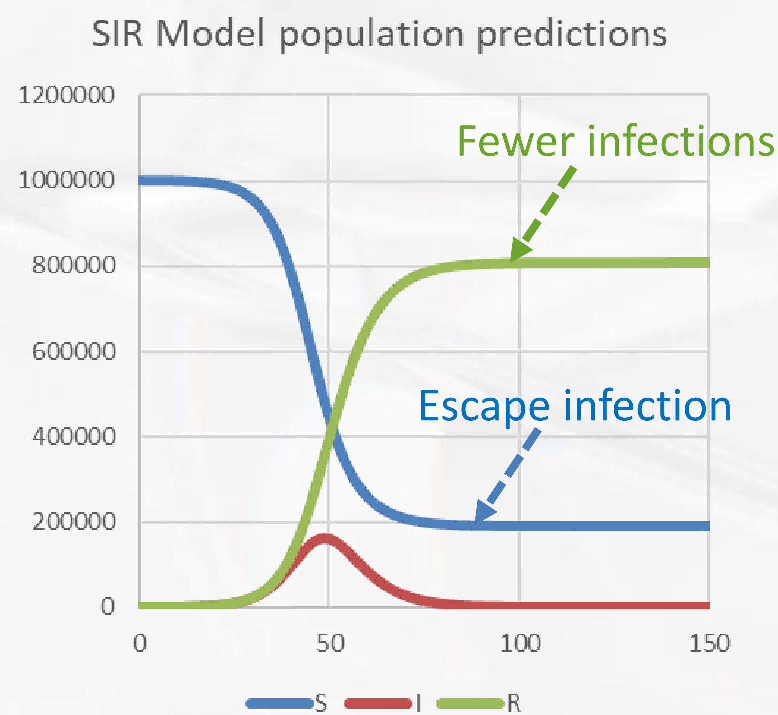
Flattening the curve with social distancing: Excel

Without distancing



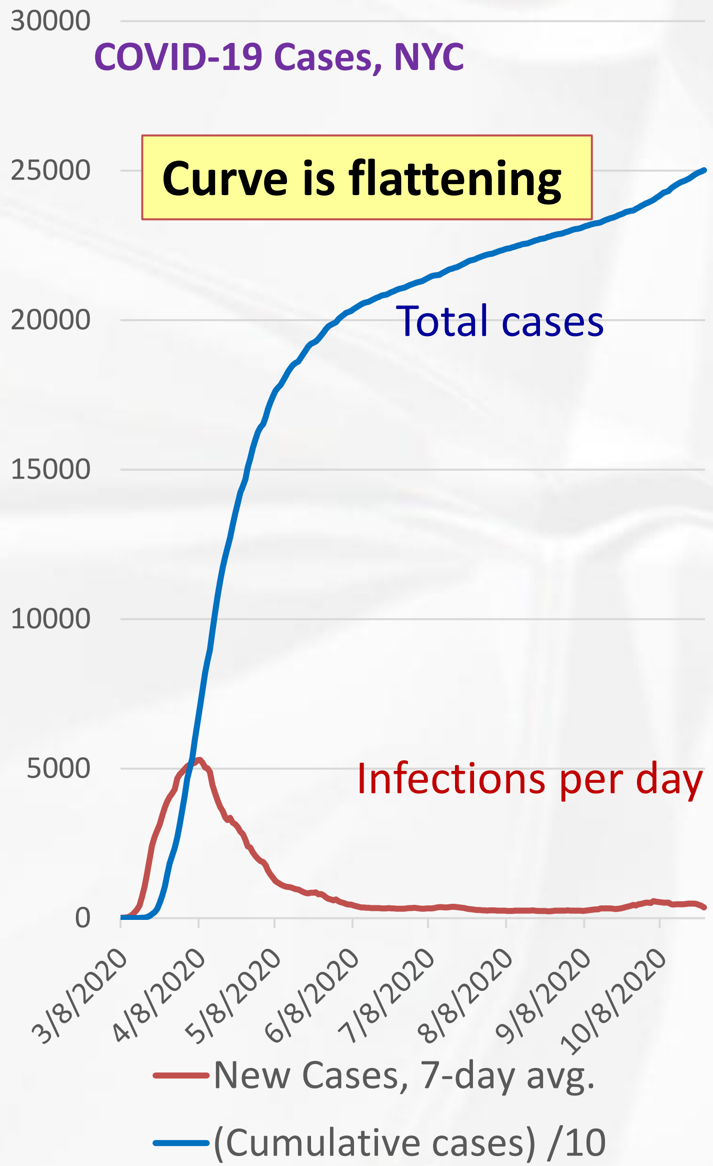
12 contacts/day,
transmission rate 0.06

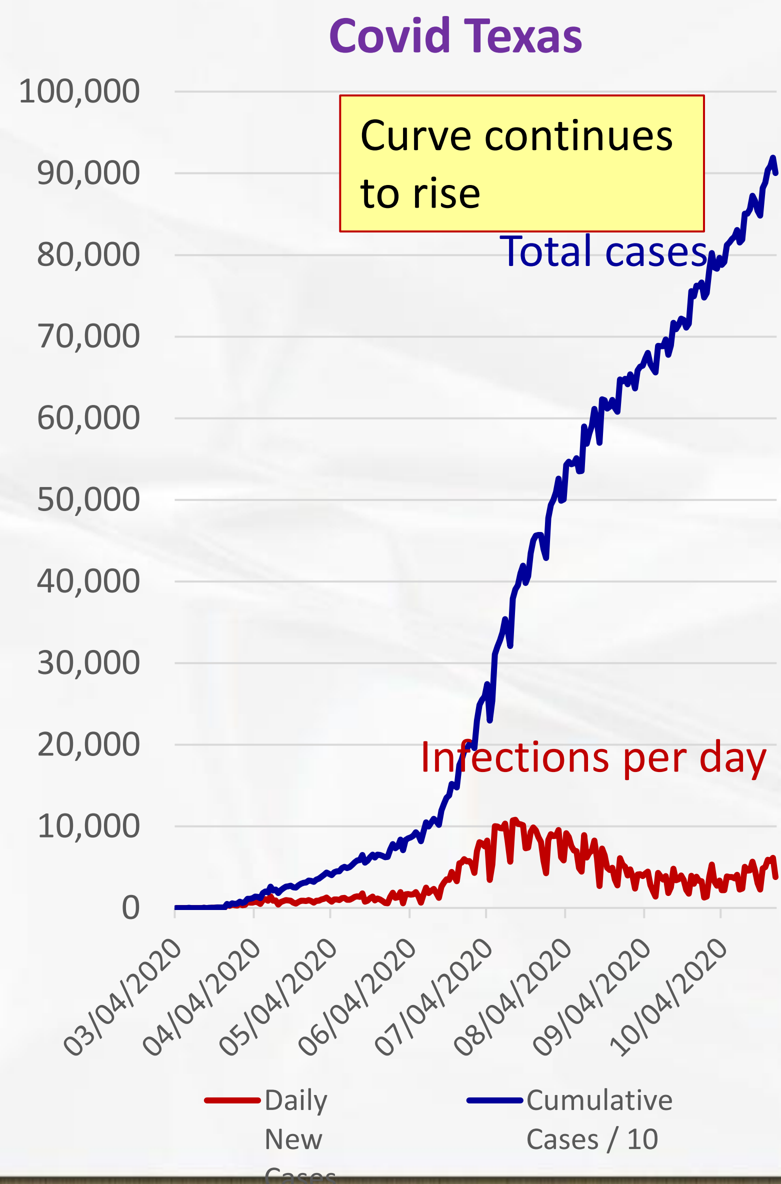
With distancing



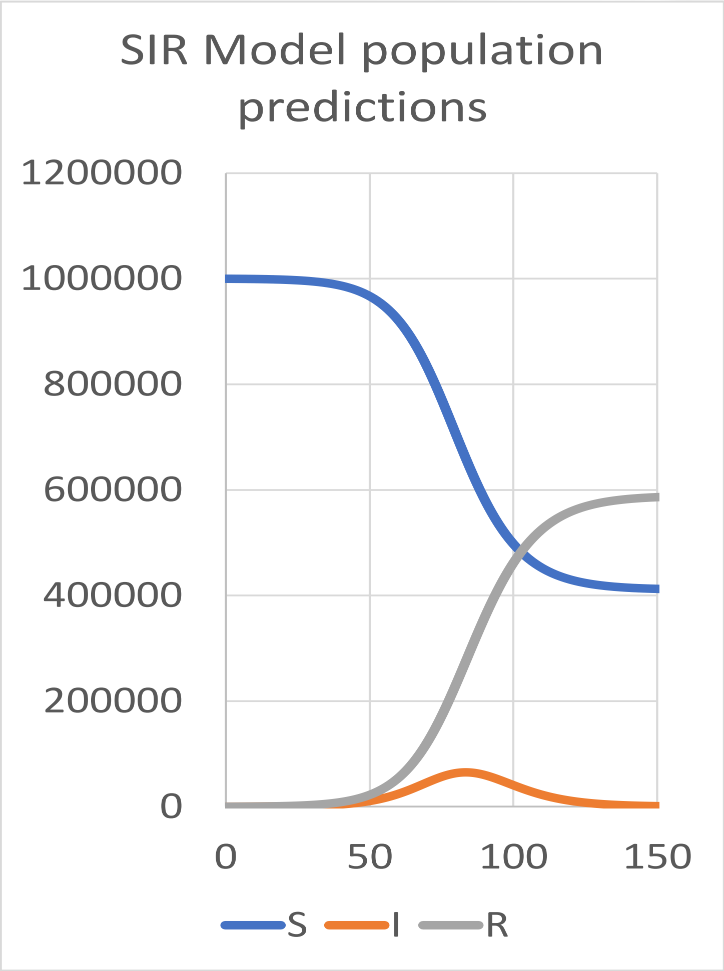
8 contacts/day,
transmission rate 0.05

Herd immunity

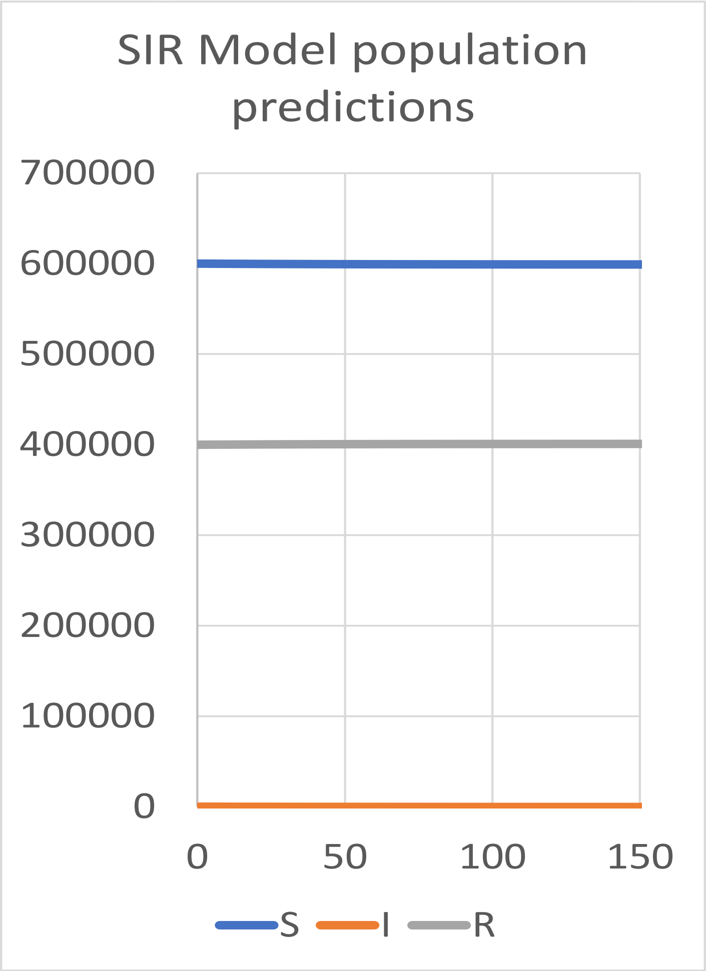




Herd Immunity—Excel demonstration



0% immune at time 0



40% immune at time 0

Basic & Effective Reproduction numbers

The **basic reproduction number (R_0)** is defined as the average number of additional infections produced by a single infection, assuming that everyone is susceptible.

The **effective reproduction number (R)** is defined as the average number of additional infections produced by a single infection in the current mixed population.

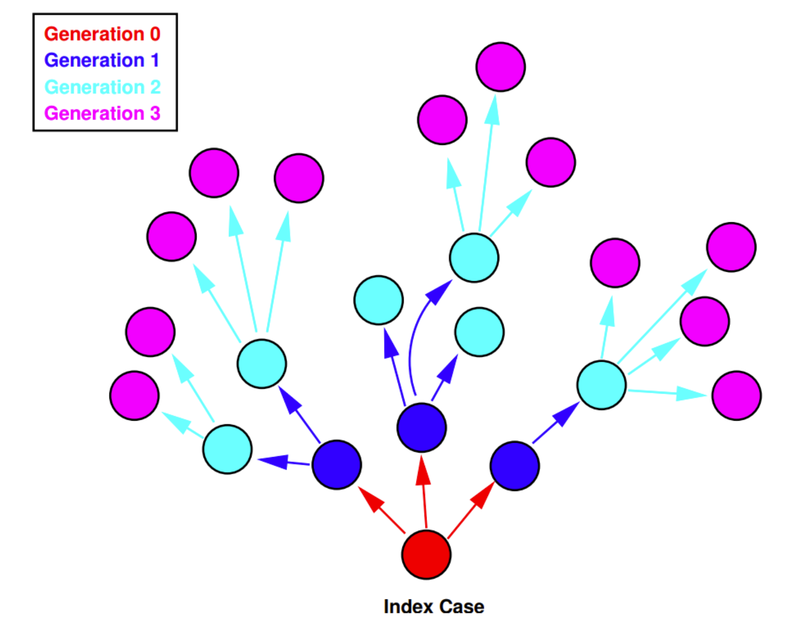
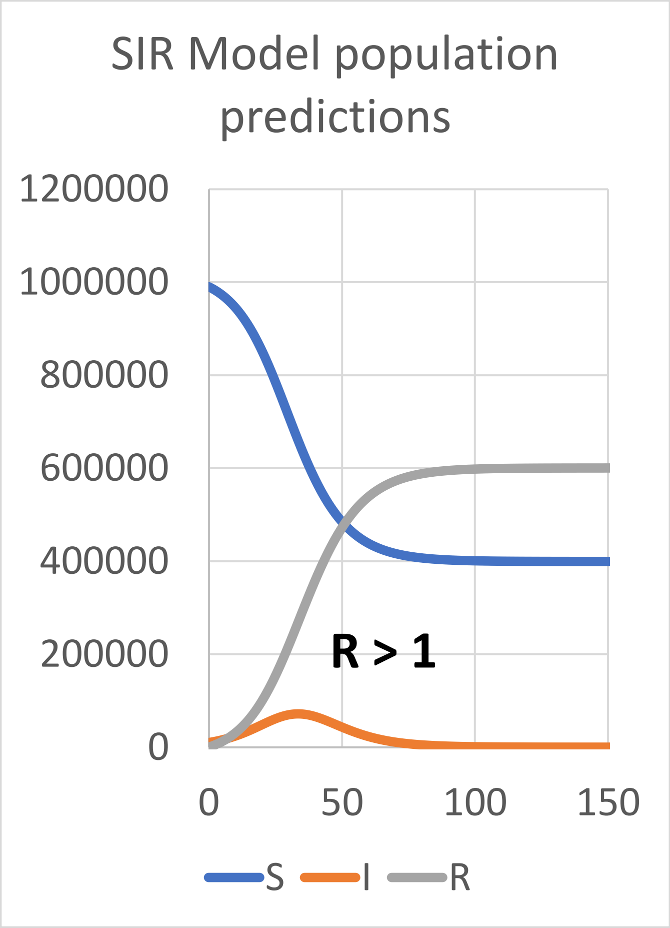


Figure 2: Graphical depiction of "generations" in an epidemic.

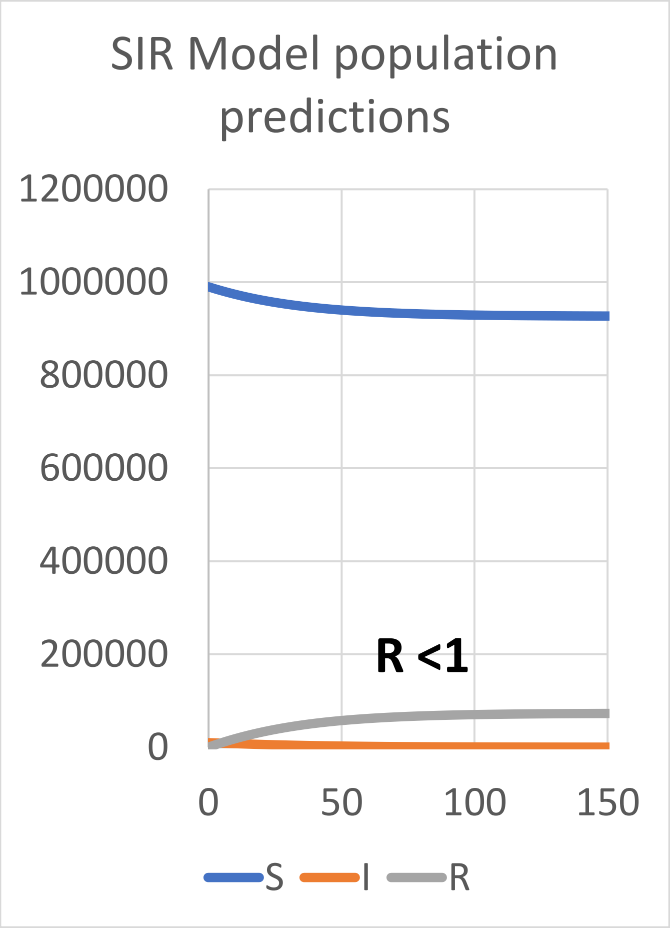
One index case has produced 12 3rd generation cases. The reproduction number is approximately $(12)^{1/3} \approx 2.2$.

https://web.stanford.edu/~jhj1/teachingdocs/Jones-on-R0.pdf

Reproduction number—Excel demonstration



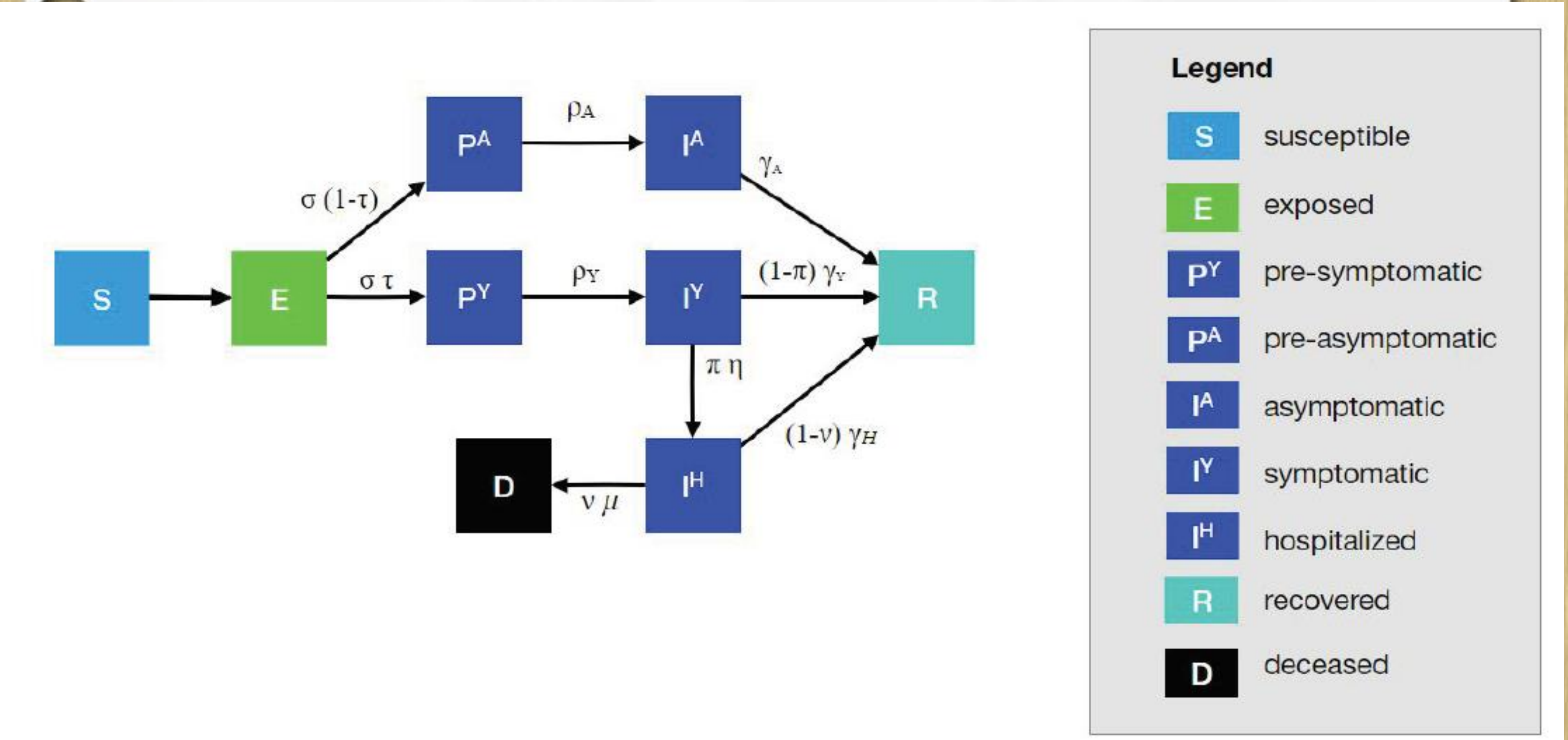
5 Contacts/day



3 Contacts/day

Enhancements I: More compartment types

- Exposed: infected, but not yet infectious
- Presymptomatic: infectious, but not yet symptomatic
- Dead: removed from population

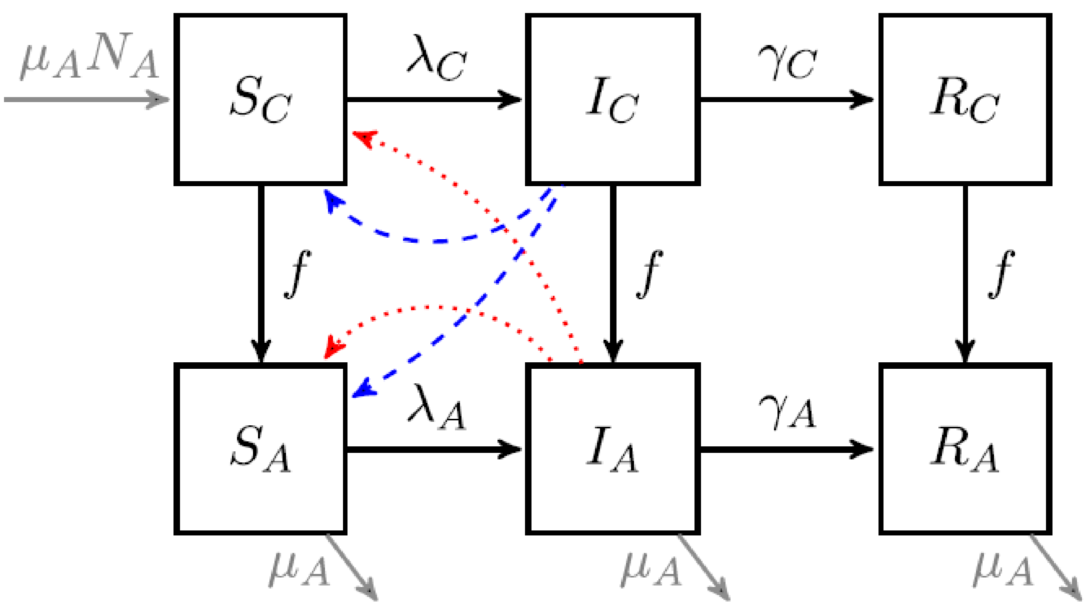


Enhancements II: stratification

Further divide population into additional compartments

Examples: age; risk (High/low/medium); location; socio-economic status; etc ...

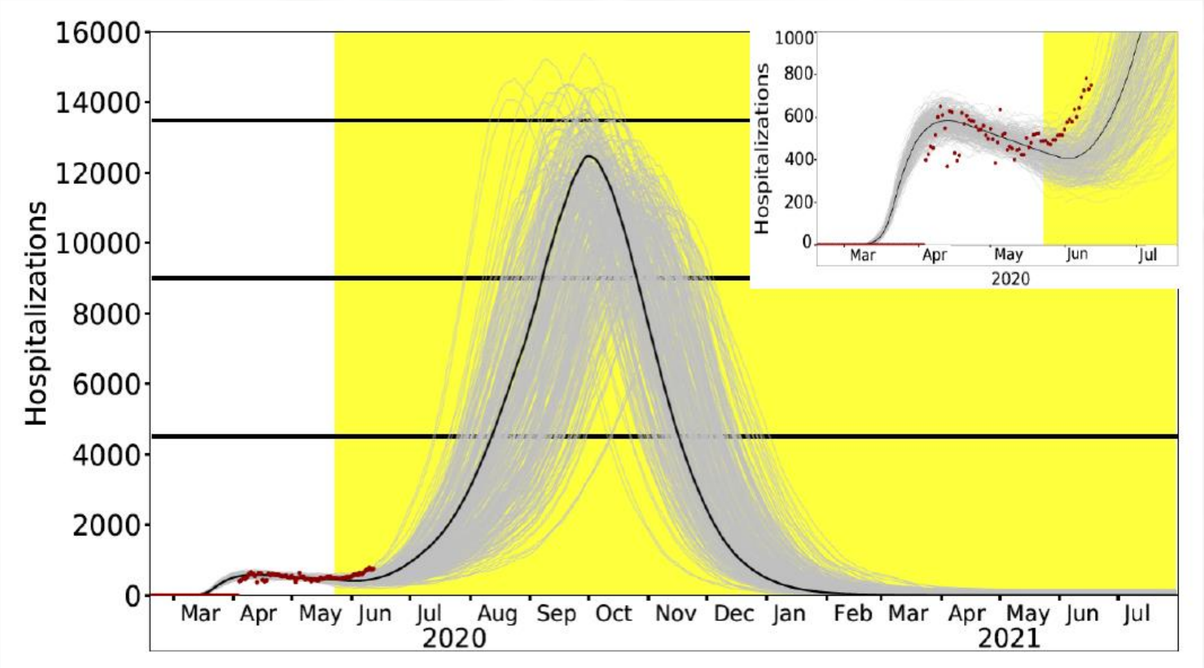
An age-stratified SIR with child (C) and adult (A) compartments. Child compartments transition to adult through aging.



Additional model enhancements

- Stochastic models
- Seasonality and time-dependence

- Graph models
- Diffusion (PDE models)
- Agent-based models



Mathematical formulation of COVID-19 epidemic model under controls

$$\frac{dS_j}{dt} = -\sum_{i=0}^{1} \frac{1}{N_i} \left(I_i^Y \omega^Y + (1 - u_i)[I_i^A \omega^A + P_i^Y \omega^{PY} + P_i^A \omega^{PA}] \right) (1 - v_j) \beta \Phi_{ji} S_j,$$

$$\frac{dE_j}{dt} = \sum_{i=0}^{1} \frac{1}{N_i} \left(I_i^Y \omega^Y + (1 - u_i)[I_i^A \omega^A + P_i^Y \omega^{PY} + P_i^A \omega^{PA}] \right) (1 - v_j) \beta \Phi_{ji} S_j - \sigma E_j.$$

$$\frac{dP_j^A}{dt} = (1-\tau)\sigma E_j - \rho^A P_j^A$$

$$\frac{dP_j^Y}{dt} = \tau \sigma E_j - \rho^Y P_j^Y$$

$$\frac{dI_j^A}{dt} = \rho^A P_j^A - \gamma^A I_j^A$$

$$\frac{dI_j^Y}{dt} = \rho^Y P_j^Y - (1 - \Pi_j)\gamma^Y I_j^Y - \Pi_j \eta I_j^Y$$

$$\frac{dI_j^H}{dt} = \Pi_j \eta I_j^Y - (1 - \nu_j)\gamma^H I_j^H - \mu \nu_j I_j^H$$

$$\frac{dR_j}{dt} = \gamma^A I_j^A + (1 - \Pi_j)\gamma^Y I_j^Y + (1 - \nu_j)\gamma^H I_j^H$$

$$\frac{dD_j}{dt} = \mu \nu_j I_j^H.$$

u_i = Testing level for $i'th$ population group

v_i = Distancing level for $i'th$ population group

Implementation cost model for testing and distancing

$$\alpha_j(u_j, N_j^A) = \begin{cases} 0 & \text{if } \; u_j = 0, \\ a_{j0} + a_{j1} N_j^A u_j + a_{j2} N_j^A (u_j)^2 & \text{if } \; 0 < u_j \le u_j^{(max)}, \end{cases}$$

$$\beta_j(v_j) = b_{j1} N_j v_j + b_{j2} N_j (v_j)^n \qquad \text{if } \; 0 < v_j \le v_j^{(max)}.$$

$$N_j^A = S_j + E_j + P_j^A + P_j^Y + I_j^A$$

no. asymptomatic individuals in subpopulation.

α_j and β_j = costs for COVID testing and social distancing respectively for subpopulations j=0,1 (low risk, high risk)

Quadratic costs reflect the *law of diminishing returns* (higher level of controls have higher marginal costs)

Calculating the reproduction number (1)

The basic reproduction number (R_0) is the average number of secondary infections produced by a typical case of an infection in a population where everyone is susceptible.

- $X = [X_0, X_1]$, $X_j = [E_j, P_j^A, P_j^Y, I_j^A, I_j^Y, I_j^H]$ (*infected classes*)
- $X' = [X'_0, X'_1]$ $X'_j = [S_j, R_j]$ (*uninfected classes*)
  $j = 0, 1$ corresponds to low and high risk subpopulations respectively.

Divide flows that affect infected classes into *inflows* and *outflows*: .

$$\frac{dX}{dt} = \mathcal{F}(\mathbb{X}) - \mathcal{V}(\mathbb{X})$$

where $\mathbb{X} = (X, X')$, $\mathcal{F}(\mathbb{X})$ = inflows, $\mathcal{V}(\mathbb{X})$ = outflows.

R_0 and R_e : Calculation (2)

$$\frac{dX}{dt} = \mathcal{F}(\mathbb{X}) - \mathcal{V}(\mathbb{X})$$

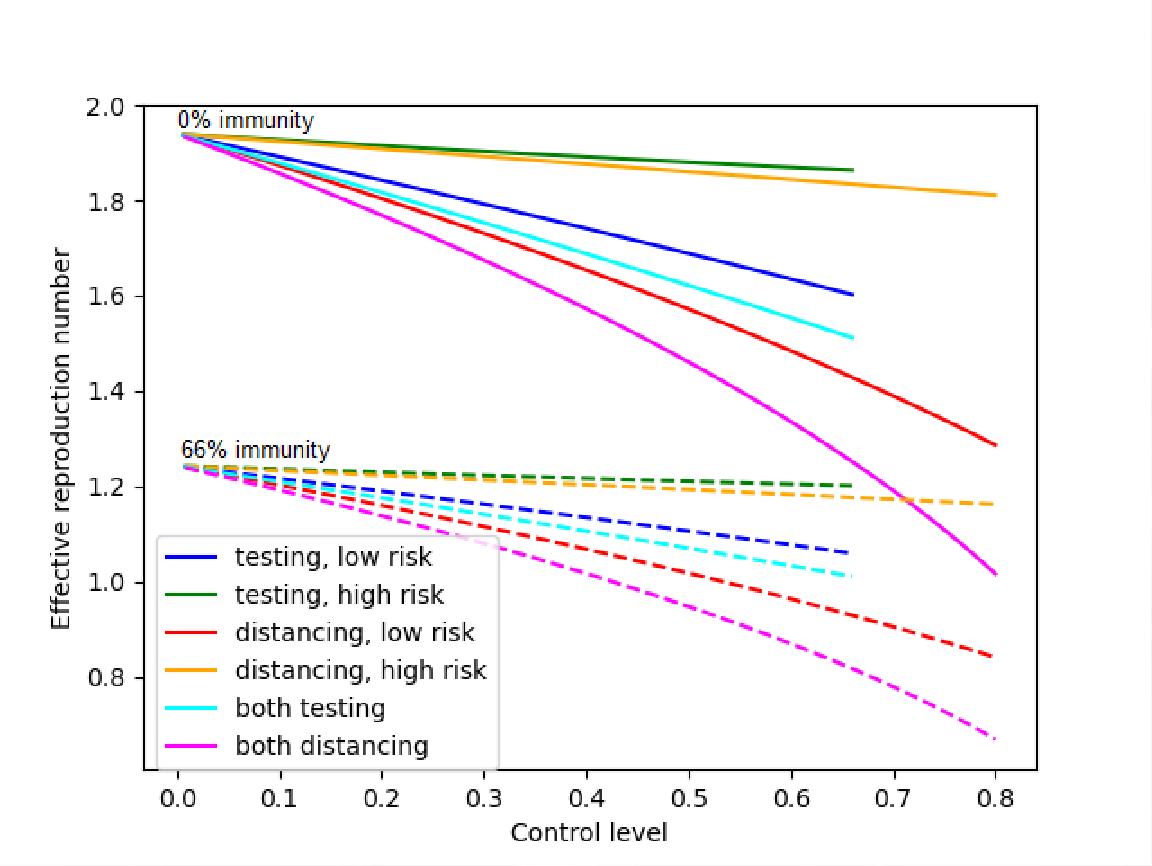
$\mathbb{X} = (X, X') =$ (infected, uninfected); $(\mathcal{F}, \mathcal{V}) =$ (inflows, outflows)
The next generation matrix is given by FV^{-1}, where

$$F = \left(\frac{\partial \mathcal{F}}{\partial X}\right)_{(0,\bar{X}')} \text{ and } V = \left(\frac{\partial \mathcal{V}}{\partial X}\right)_{(0,\bar{X}')}.$$

The spectral radius (largest eigenvalue) of the matrix FV^{-1} is called the *basic reproduction number* (denoted by R_0).

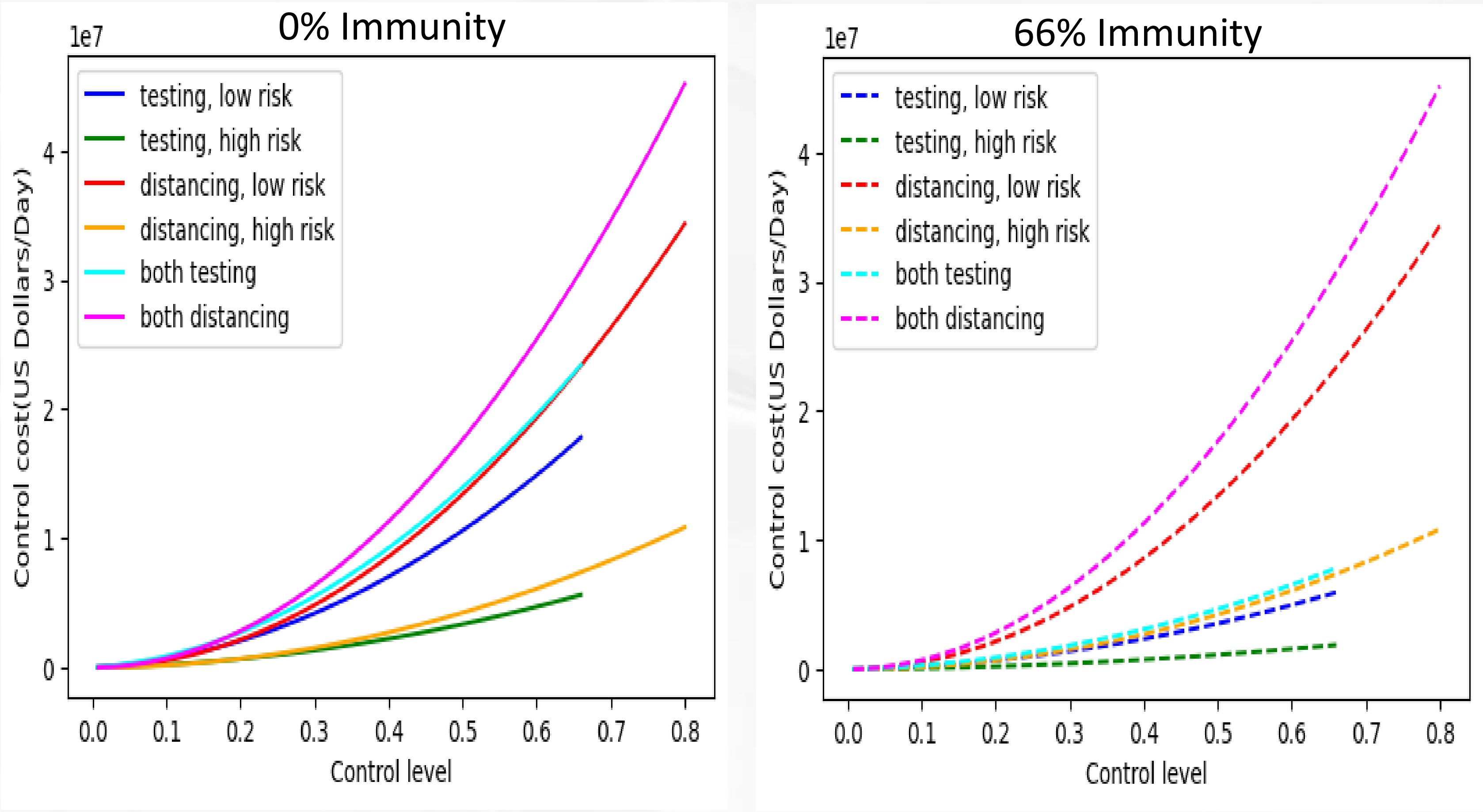
When $(0, \bar{X}')$ is replaced by the current state and $\mathcal{F}$ is modified to include the effects of testing and distancing, then the spectral radius of FV^{-1} gives the *effective reproduction number* R_e.

Results (1)



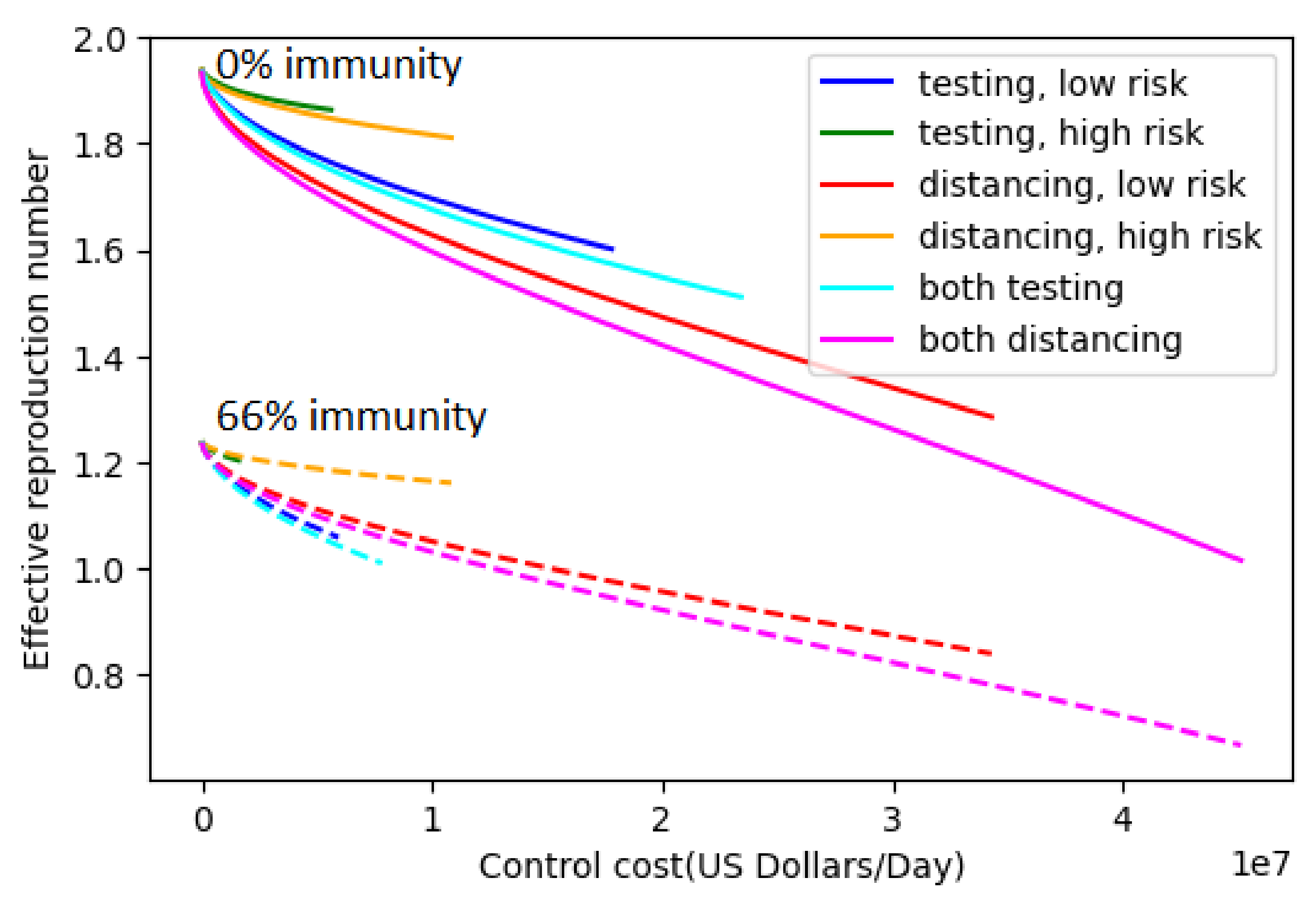
- Distancing has a greater effect than testing
- Controls on entire population have greater effect than controls on subpopulations

Results (2)



- Control costs for testing shift are relatively lower for higher levels of immunity

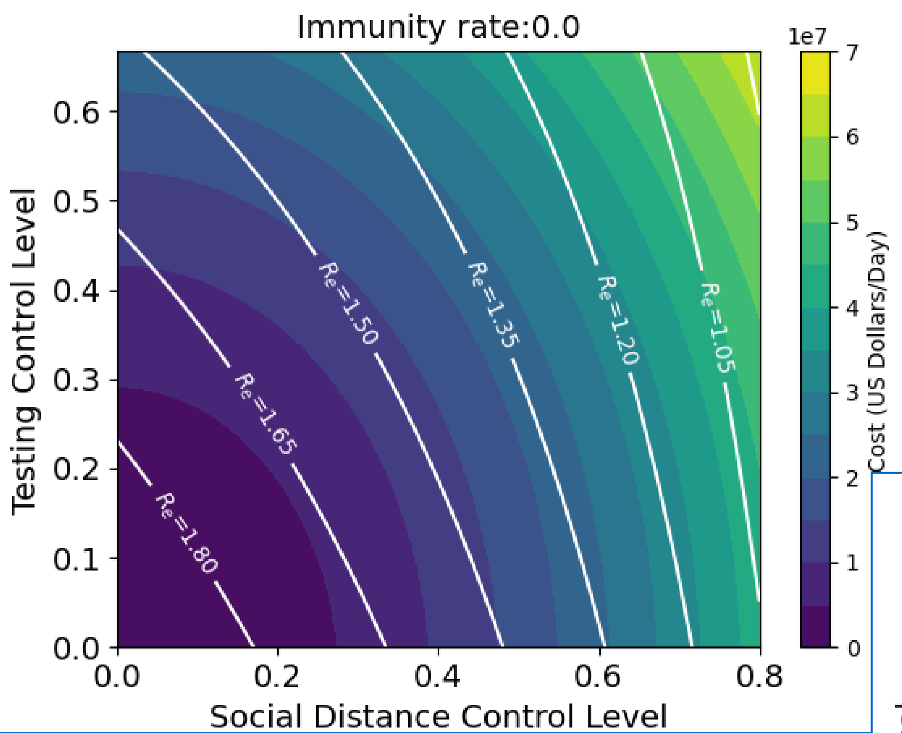
Results (3)

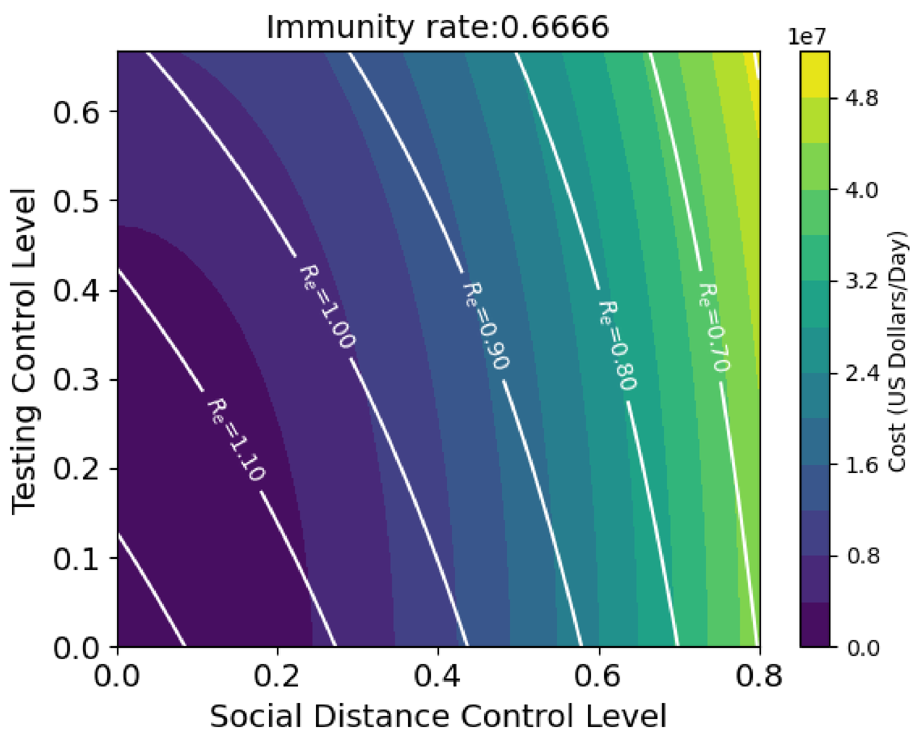


Most cost-effective strategy:

- higher levels of immunity—distancing of entire population
- lower levels of immunity—testing of entire population

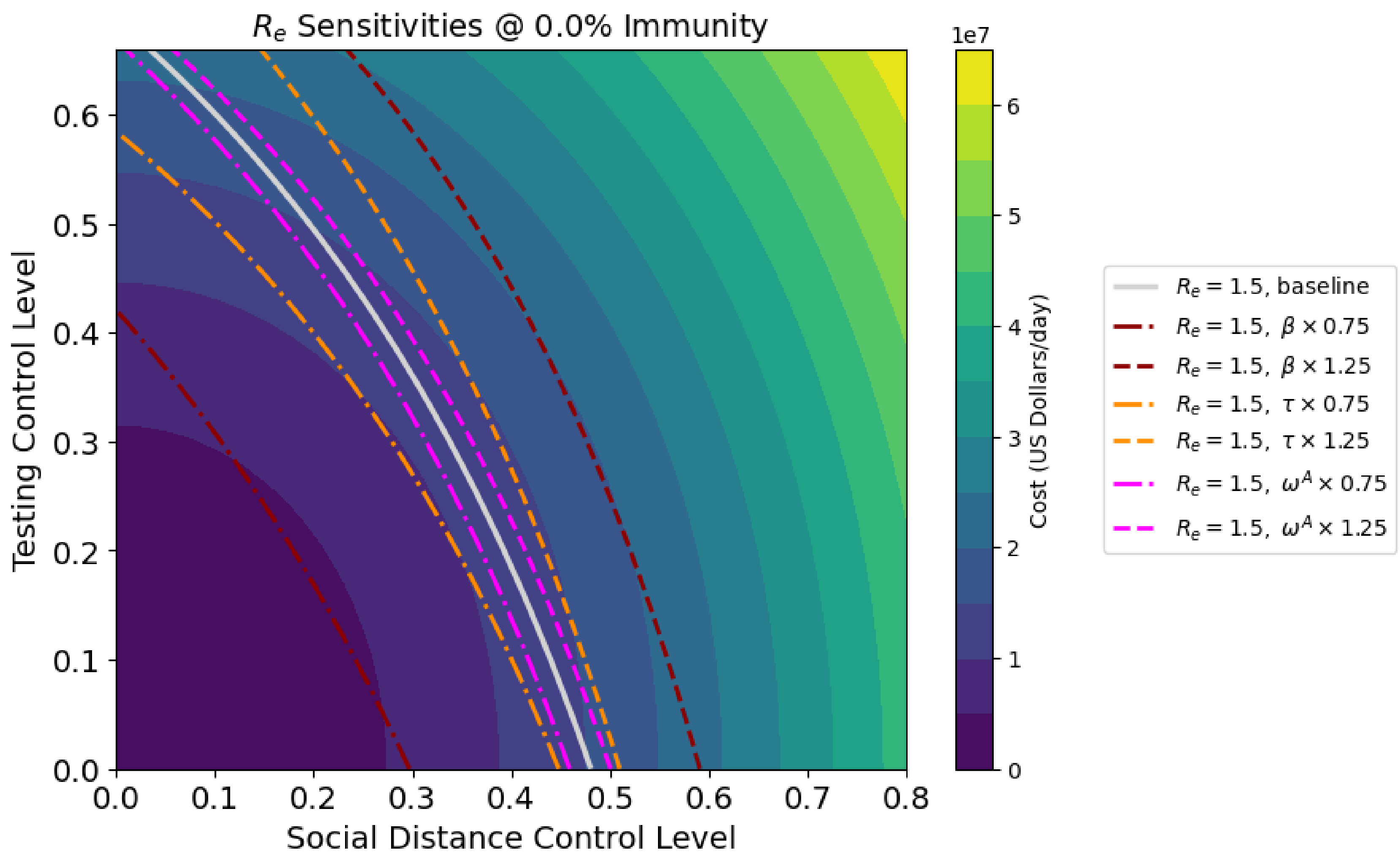
Results (4)





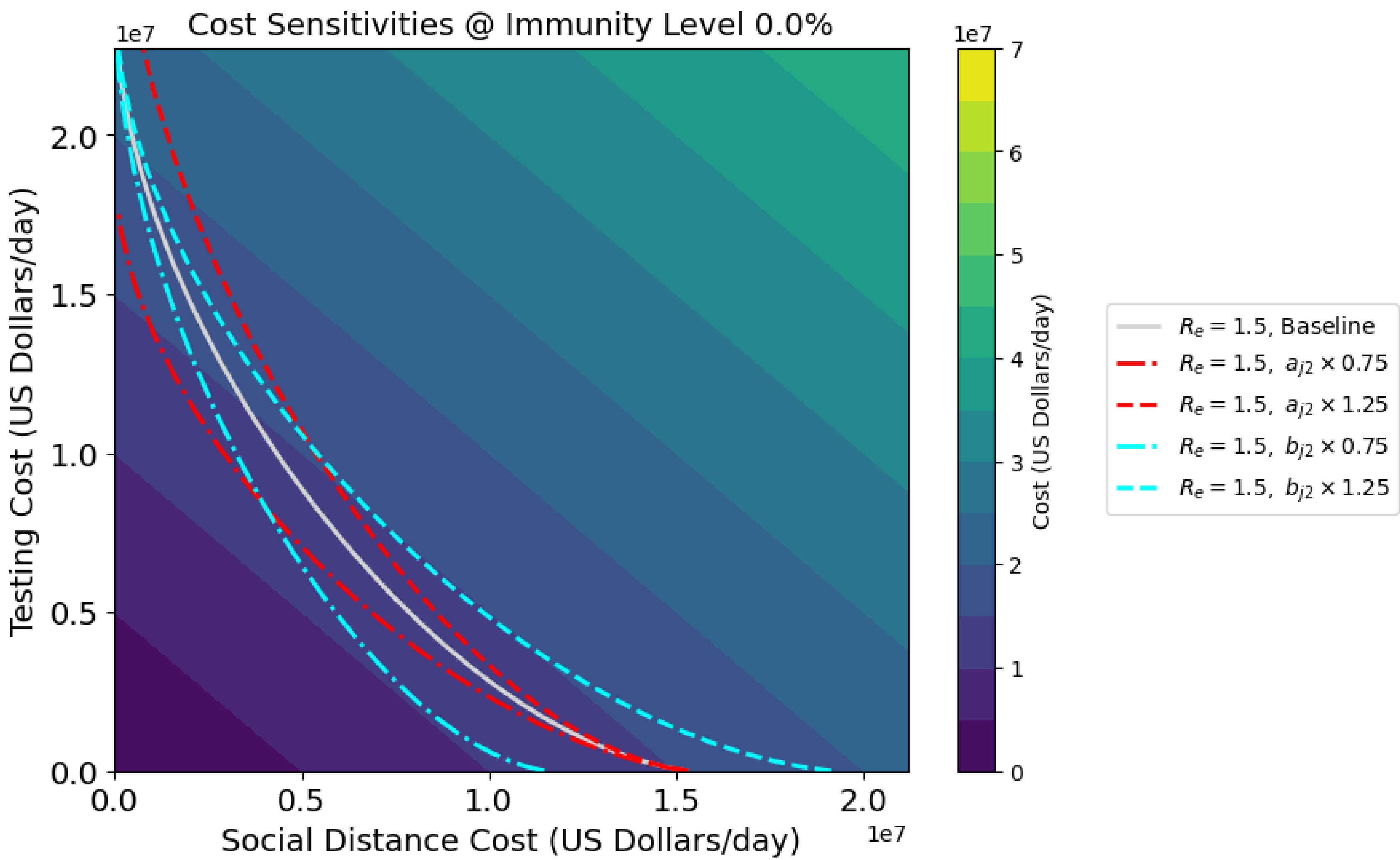
- similar tradeoffs between cost and effective reproduction number at different levels of immunity

Results (5)



- Cost of reducing reproduction number is highly sensitive to model parameter values

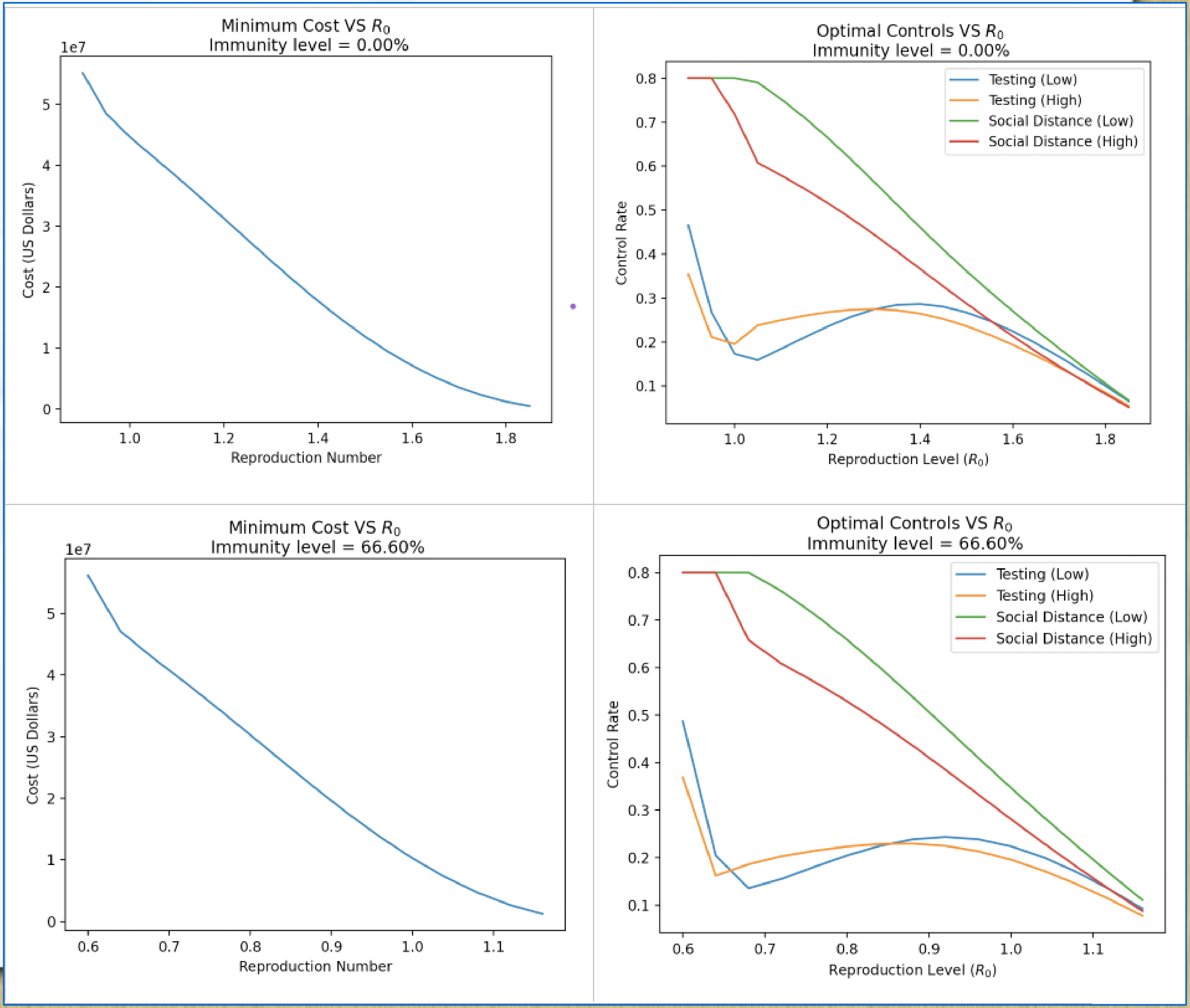
Results (6)



- Cost of reducing reproduction number is highly sensitive to cost parameter values

Results (6)

- Optimal control mix scales for different immunity levels



Resources

The SIR Model for Spread of Disease – Introduction (*Mathematical Association of America)*
https://www.maa.org/press/periodicals/loci/joma/the-sir-model-for-spread-of-disease-introduction

An introduction to compartmental modeling for the budding infectious disease modeler
Letters in Biomathematics 5(1):195-221
https://www.researchgate.net/publication/327067041_An_introduction_to_compartmental_modeling_for_the_budding_infectious_disease_modeler

Thank you for your attention

Mo dupe fun ifetisile re

شكرا على الاهتمام

Merci pour votre attention

感谢您的关注

ϲєпϨмот

Asante kwa mawazo yako

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Ngiyabonga ukulalela kwenu

Enkosi ngosinaka ixehsa lakho

thron@tamuct.edu

UCD ENERGY INSTITUTE

IORMS
Institute of Operational Research and Management Science of Nigeria

Sustainability Modelling and Optimisation - Contributions to affordable and clean energy

Dr. Paula Carroll,
University College Dublin, Ireland

UCD DUBLIN
UCD College of Business

UCD ENERGY INSTITUTE

UCD DUBLIN UCD College of Business

Outline

1. Energy System Transformation
2. Optimisation & Data Science Problems:
   1) Unit Commitment;
   2) Wind Power Prediction;
   3) Smart Grid Topology Design;
   4) Heat Pump Efficiency Management;
   5) Green Vehicle Routing Problems;
   6) Smart Meter Data Classification.
3. Summary

Grand Societal Challenges

Sustainable Development Goal 5

Achieve gender equality and empower all women and girls

Sustainable Development Goal 7

Ensure access to affordable, reliable, sustainable and modern energy for all

Sustainable Development Goal 13

Take urgent action to combat climate change and its impacts

Grand Societal Challenges are not new: “*Who of us would not be glad to lift the veil behind which the future lies hidden; to cast a glance at the next advances of our science and at the secrets of its development during future centuries*?”…….Hilbert (1902)

The Energy Trilemma

Paris Agreement signatories aim to keep temperature increase to $<1.5^{\circ}$.
The big challenge is how to do that in a manner that is:

1. Secure;
2. Equitable;
3. Sustainable.

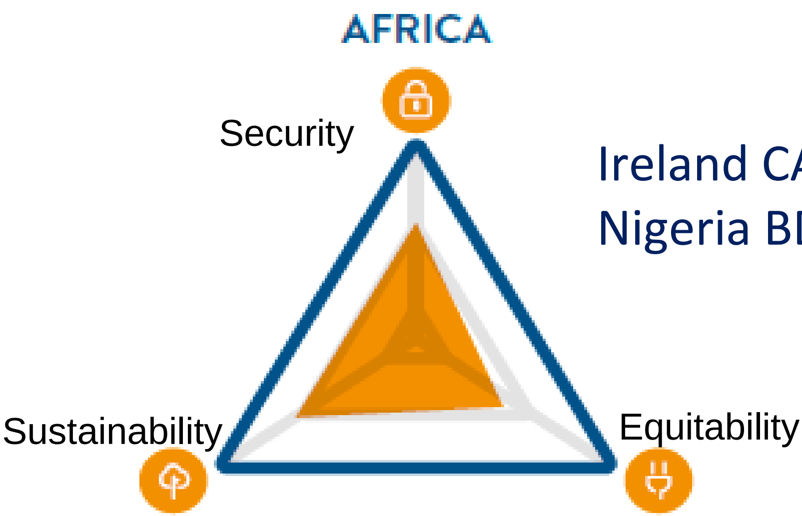
Economies rely on energy to power industry, heat and cool homes and businesses, transport people and goods.
Unfortunately worldwide energy production relies on fossil fuels.
EU countries create Strategic Energy Technology Plans (SET Plans) to transition towards a climate neutral energy system through the development of low-carbon technologies. Ireland is a member of the EU with a target for 70% of its electricity to be produced from renewables by 2030[1].

New rules for EU Electricity Market, approved May 2019:
"Consumers will be able to participate actively, individually or through communities, in all markets, either by generating electricity and then consuming, sharing or selling it, or by providing storage services.

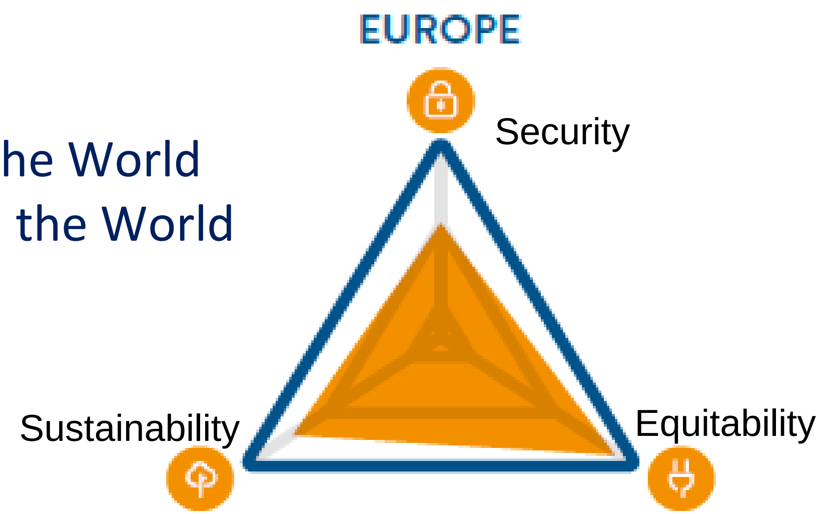
1 DCCAE (2019). Climate action plan 2019. Technical report, Department of Communications, Climate Action & Environment, Government of Ireland.

The Energy Trilemma

The World Energy Trilemma Index is a rating of national energy system performance across three dimensions: Energy Security, Energy Equity and Energy System Environmental Sustainability.



Ireland CAA 25th in the World
Nigeria BDD 125th in the World



AFRICA

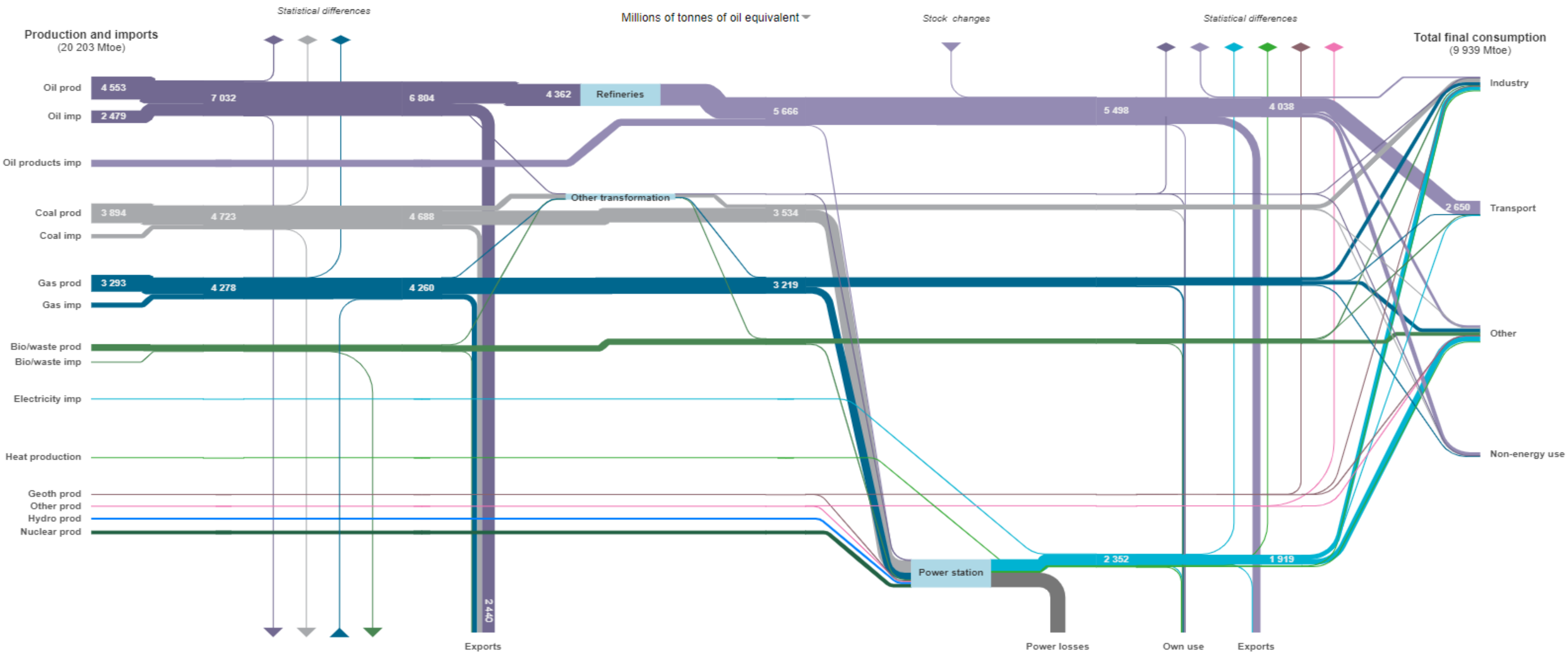
PROGRESS TOWARDS SUSTAINABLE ENERGY FOR ALL NEEDS IMPROVED INSTITUTIONAL CAPABILITIES

EUROPE

TRILEMMA CHALLENGES OF ADVANCED TRANSITION

https://trilemma.worldenergy.org/reports

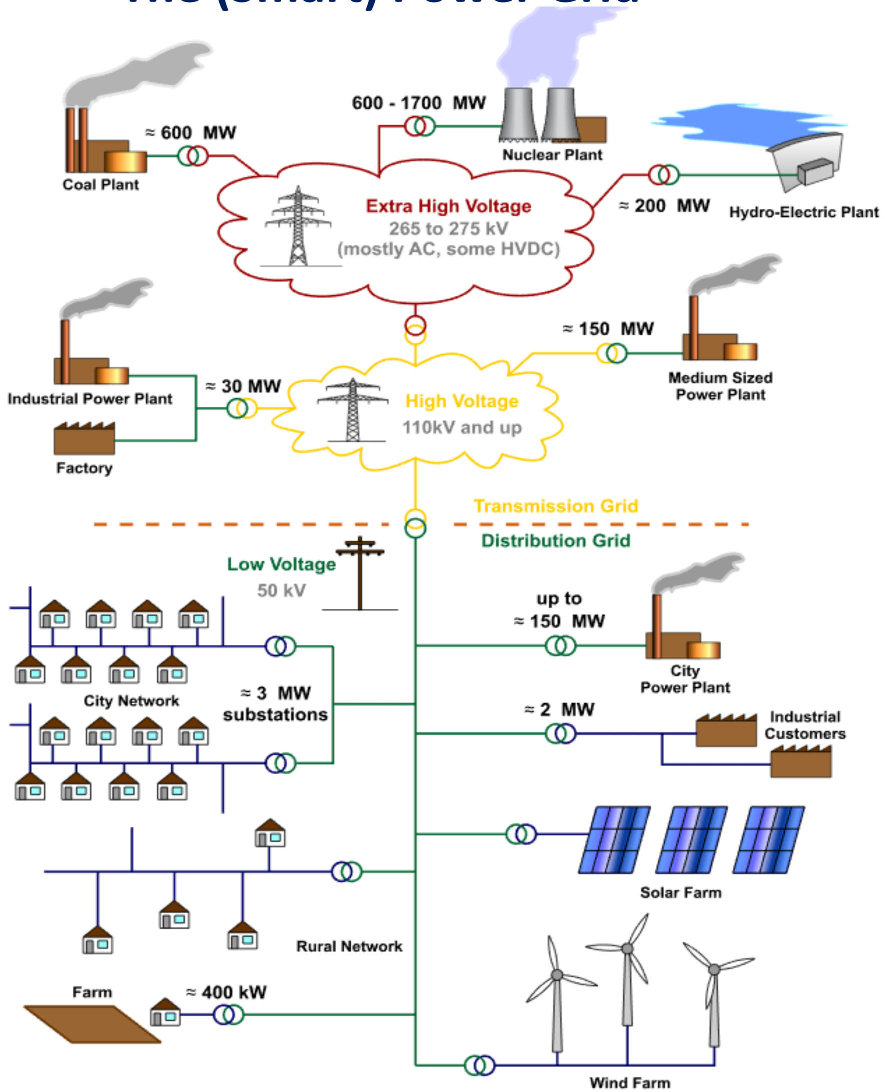
The Energy Balance Problem



https://www.iea.org/Sankey/

Optimisation and Data Science Problems

The (Smart) Power Grid



Source: https://en.wikipedia.org/wiki/Electrical_grid

Unit Commitment Problems

Data and Parameters

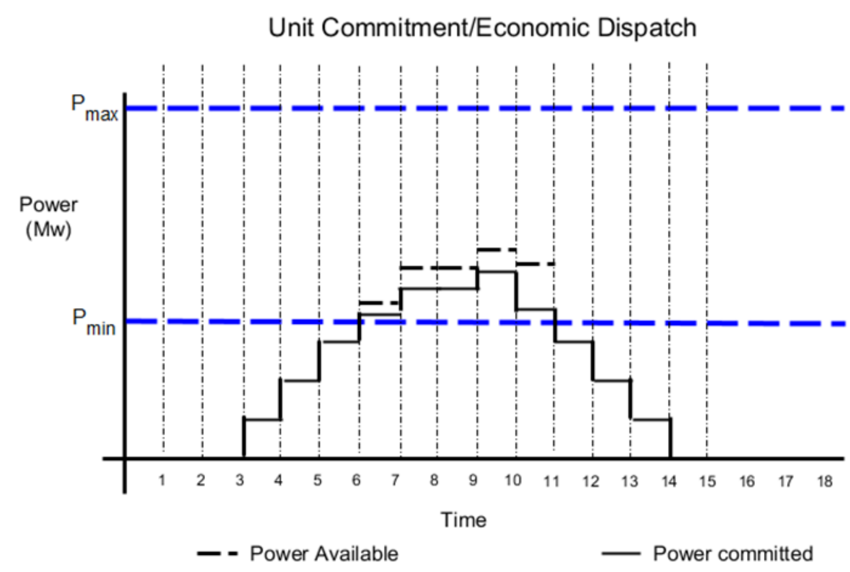
Given an electricity system:

- A set of generator units (GU) ***G*** and operating characteristics;
- A set of estimated demands ***D*** and required reserves ***R*** per time-step ***k*** over a planning horizon of ***K*** time periods.

Problem:

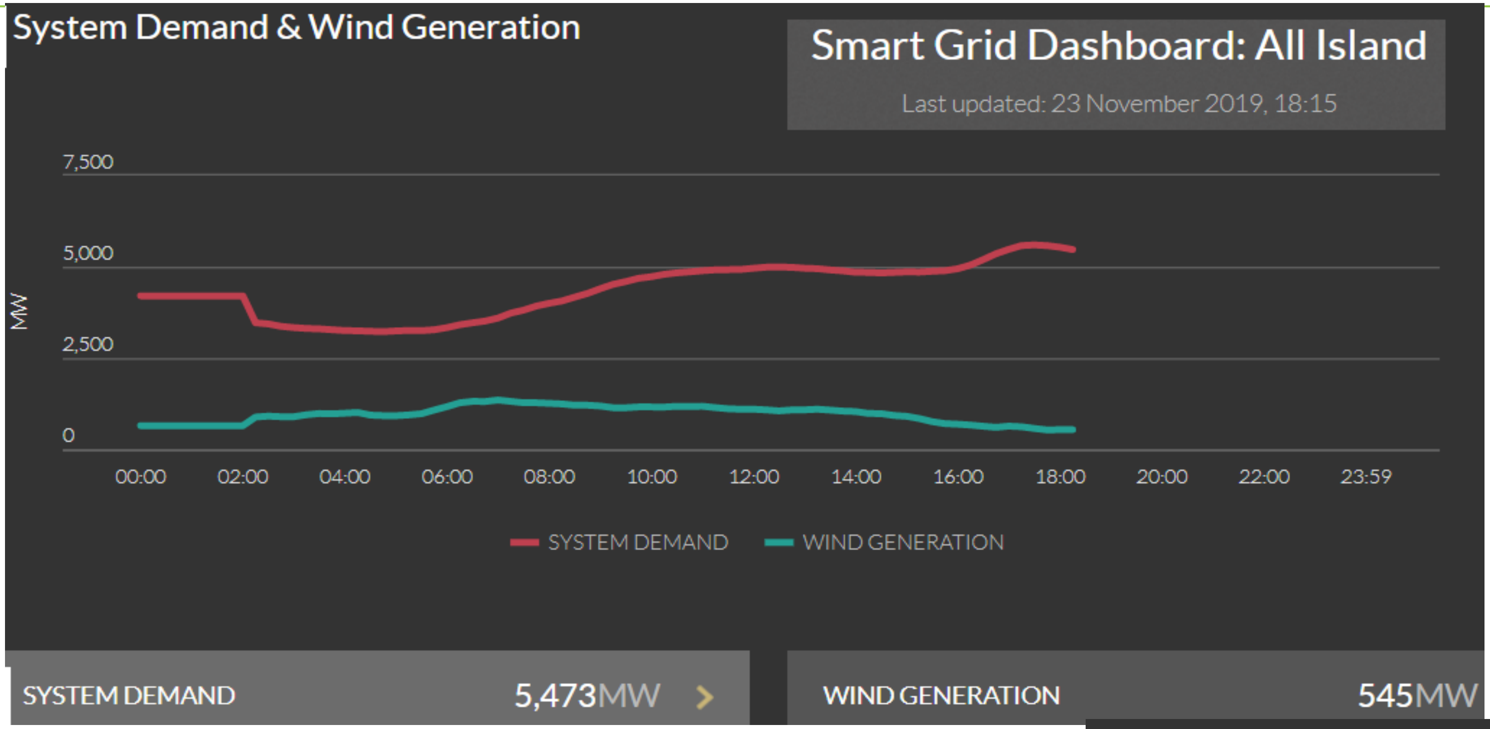
Determine the minimum cost GU dispatch schedule that satisfies demand and operational constraints.

Note: The UC problem is NP-Hard

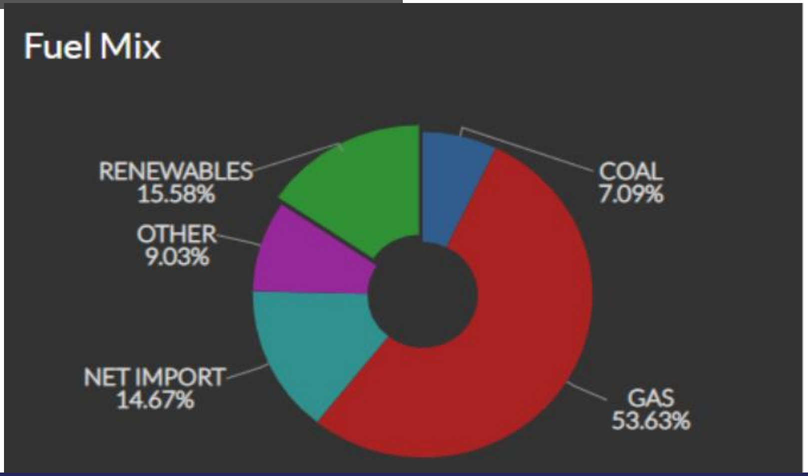


Carroll, P., Flynn, D., Fortz, B., & Melhorn, A. (2017, July). Sub-hour unit commitment MILP model with benchmark problem instances. In International Conference on Computational Science and Its Applications (pp. 635-651). Springer.

Unit Commitment Problems



http://smartgriddashboard.eirgrid.com/



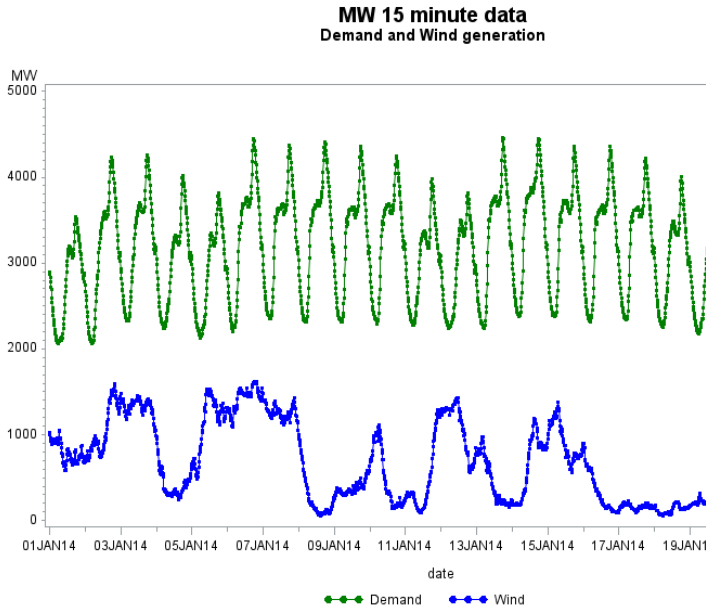
Unit Commitment Problems

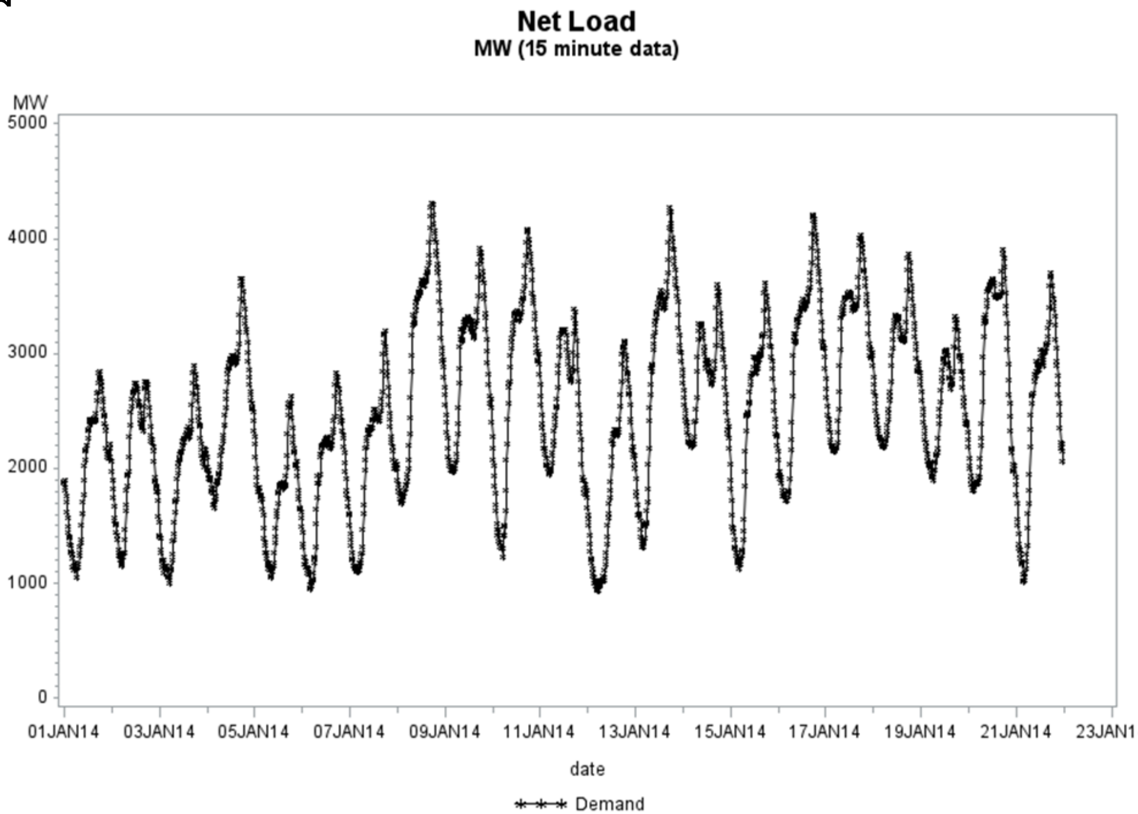
Irish_54 test system, net sub-hourly demand 2014

Sharper ramping events;

Low net load:

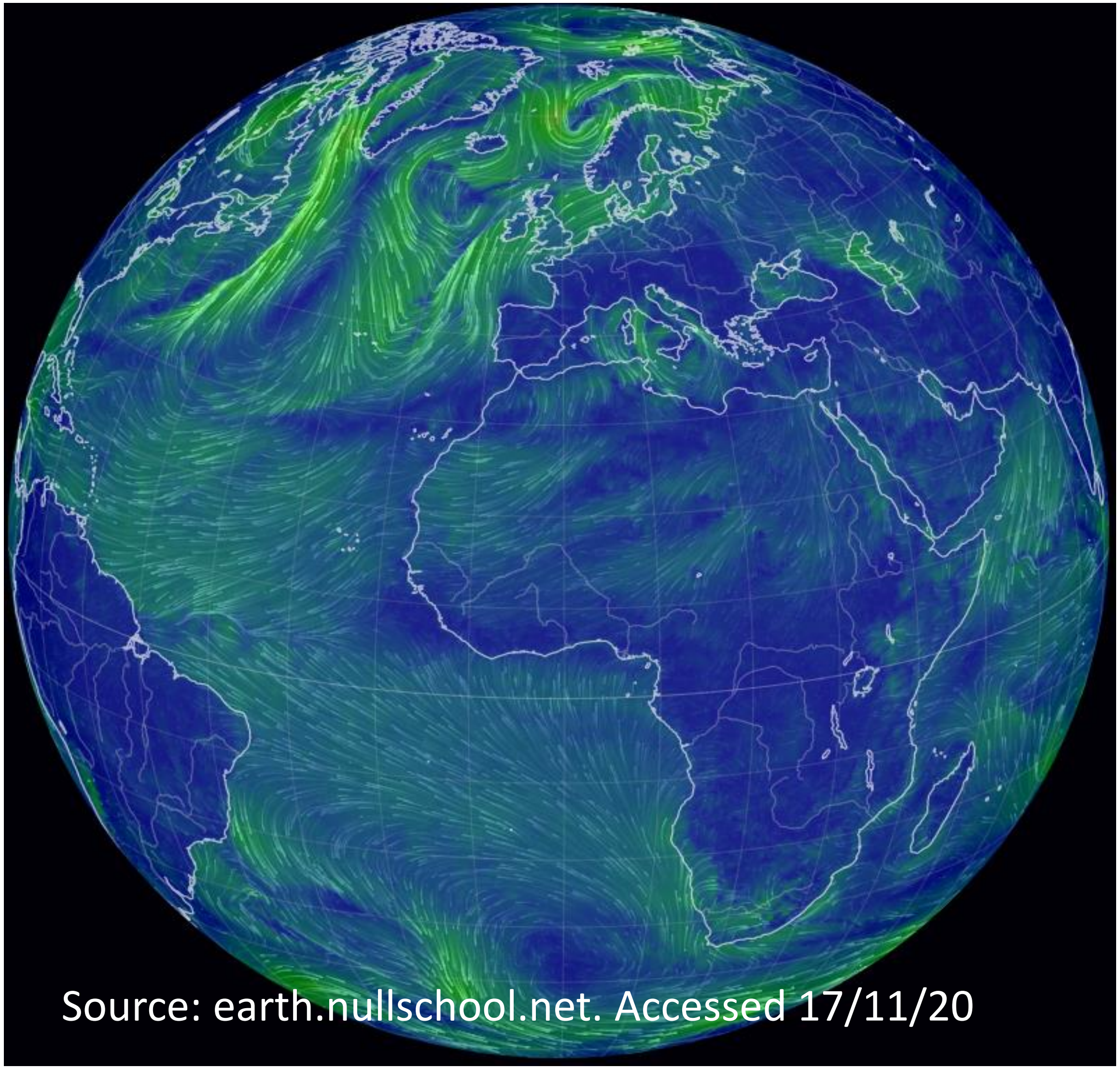
Revised reserve rules required





Eirgrid data

Wind Power Prediction Problems



Source: earth.nullschool.net. Accessed 17/11/20

Wind Power Prediction Problems

Data and Parameters

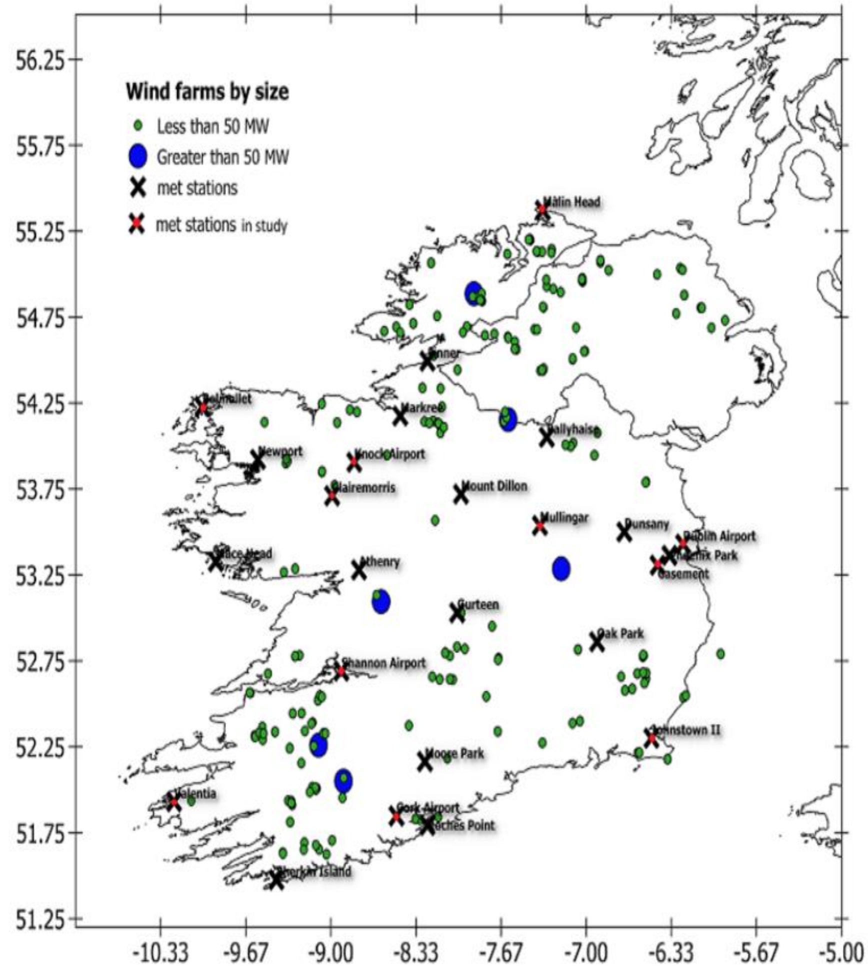
Wind Speed (knots) at 10 m height 2005 – 2015 at meteorological stations in Ireland

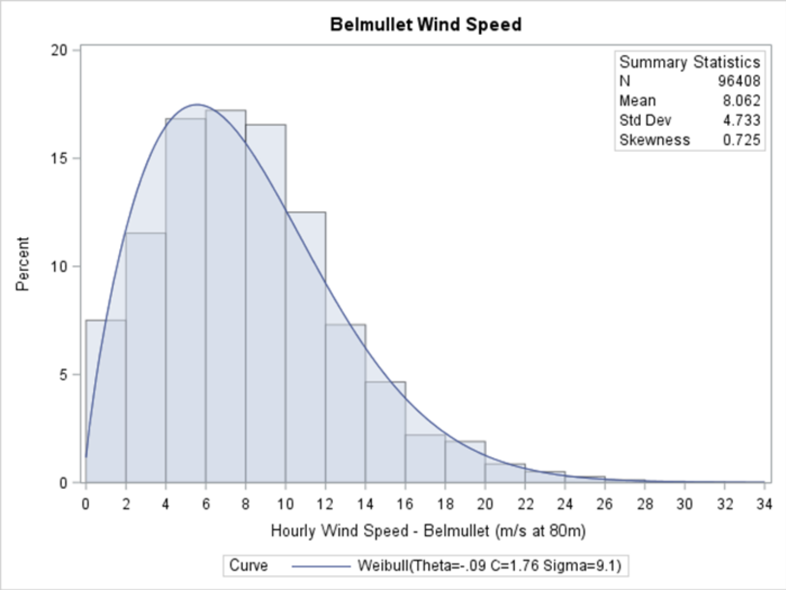
Problem Statement

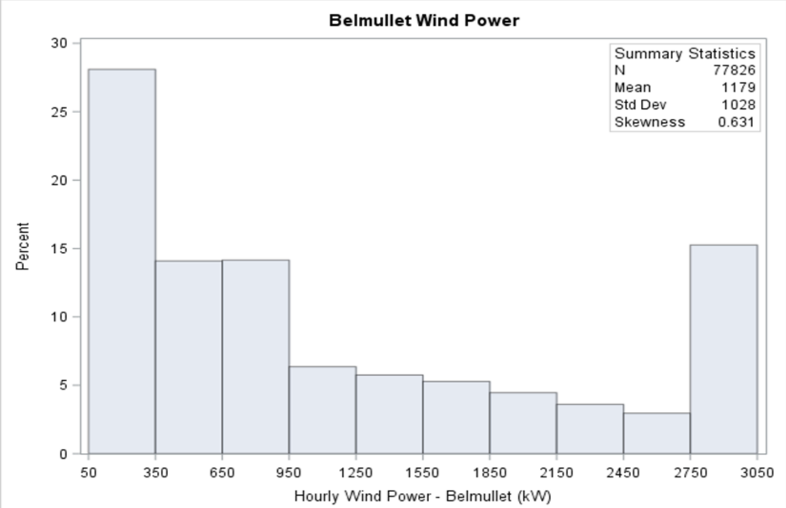
Which statistical models are useful to explain the duration of wind power events?

Carroll, P., Cradden, L. C., & Ó hÉigeartaigha, M. (2018). High Resolution Wind Power and Wind Drought Models. International Journal of Thermal and Environmental Engineering, 16(1), 27-36.

Wind Power Prediction Problems





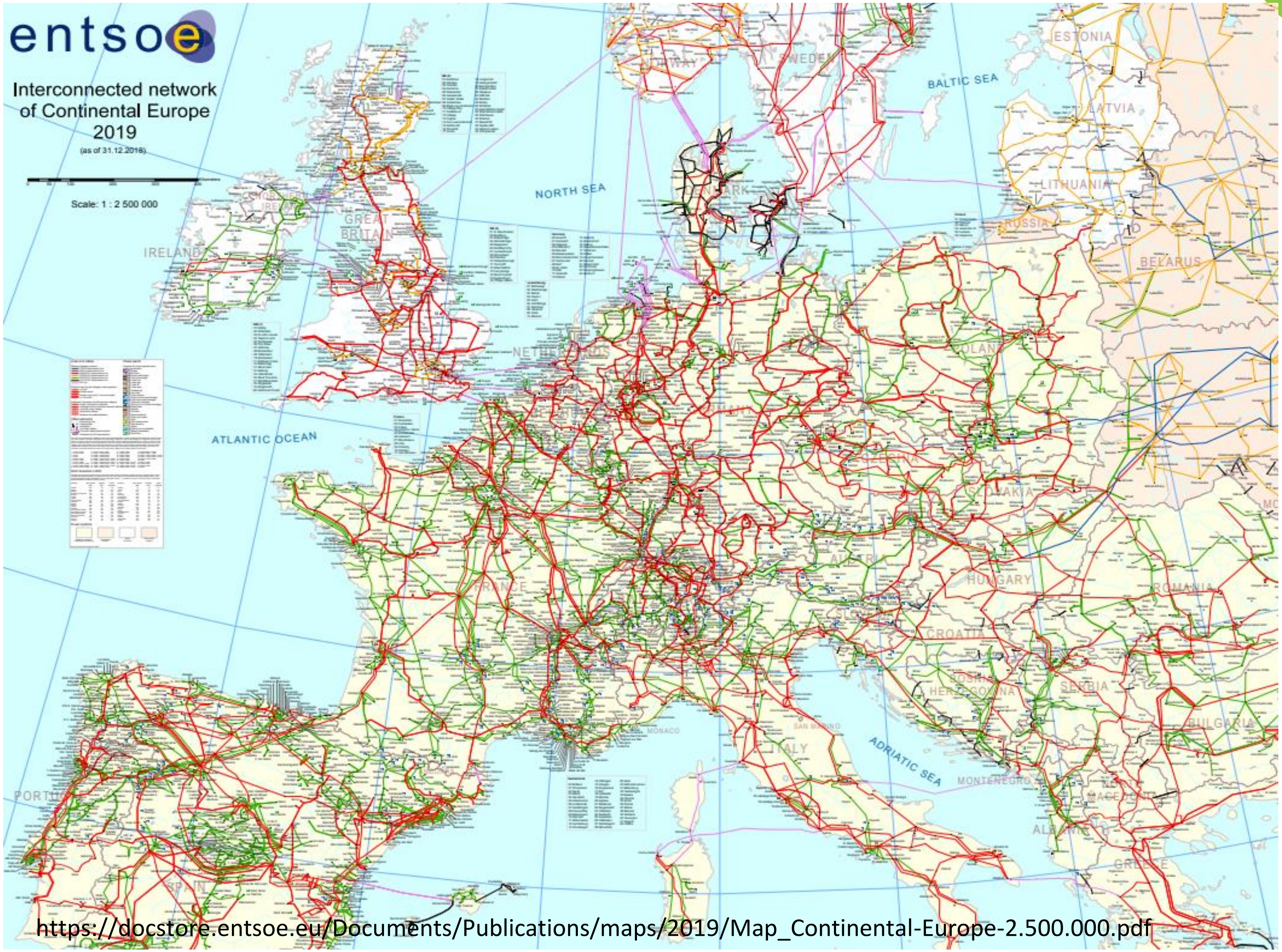


Wind Power Prediction Problems

The Models for *X* hours of power per wind event:

- Zero Truncated Poisson
  - $P(X=x) = \begin{cases} \theta & x = 0 \\ (1-\theta)\frac{P(X=x)}{1-P(x=0|\lambda)} = (1-\theta)\frac{\lambda^x e^{-\lambda}}{(1-e^{-\lambda})x!} & x \geq 1 \end{cases}$
  - Mixed model = Hurdle + Zero Truncated Poisson;
  - Equi-dispersion assumption doesn't match observed heavy tail.
- Zero Truncated Burr
  - $f(x) = ck\frac{x^{c-1}}{(1+x^c)^{k+1}}$ | x > 0
  - Mixed model = Hurdle + Burr, *c* and *k* are shape parameters

Grid Topology Problems



https://docstore.entsoe.eu/Documents/Publications/maps/2019/Map_Continental-Europe-2.500.000.pdf

Smart Grid Topology Design

Data and Parameters

An existing electricity distribution network $G=(N,A)$, where G is a tree rooted at a substation.

An energy community who wish to be as self-sufficient as possible.

The energy community represented by the nodes N of G

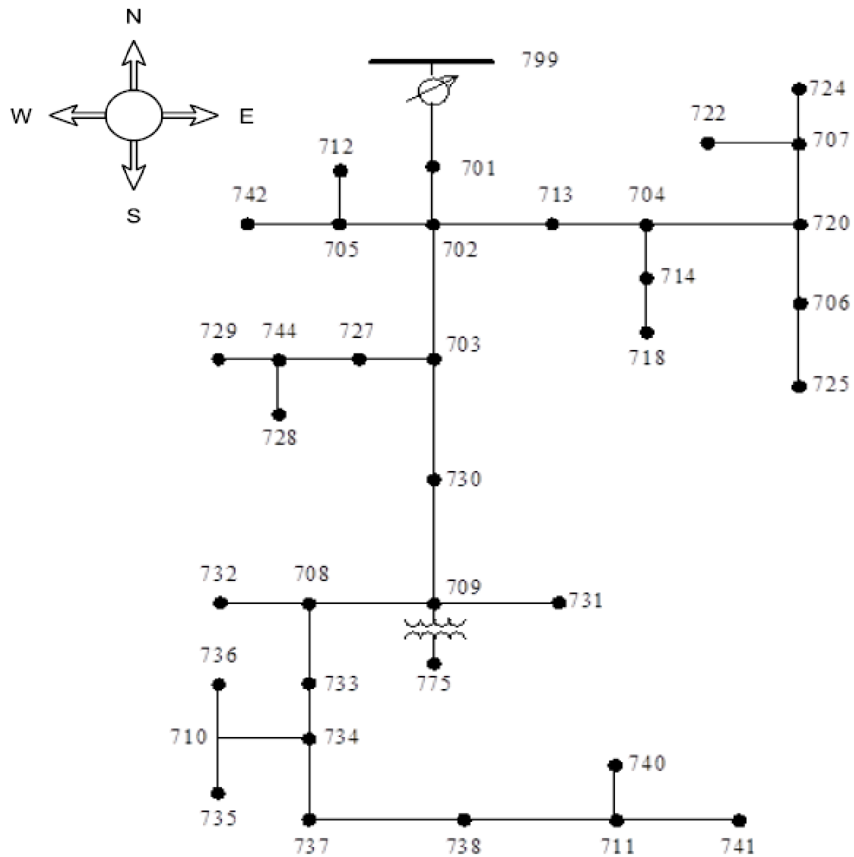
N can be partitioned into C, traditional end users, and P, a set of prosumers who produce and consume electricity.

Estimated electricity demand and renewable energy sources (RES).

Problem Statement

How should G be augmented to support electricity flows within the energy community?

Smart Grid Low Voltage Network Upgrade



Existing network is a tree distribution network *G*=(*N*,*A*)

$$N = n_0 + C$$

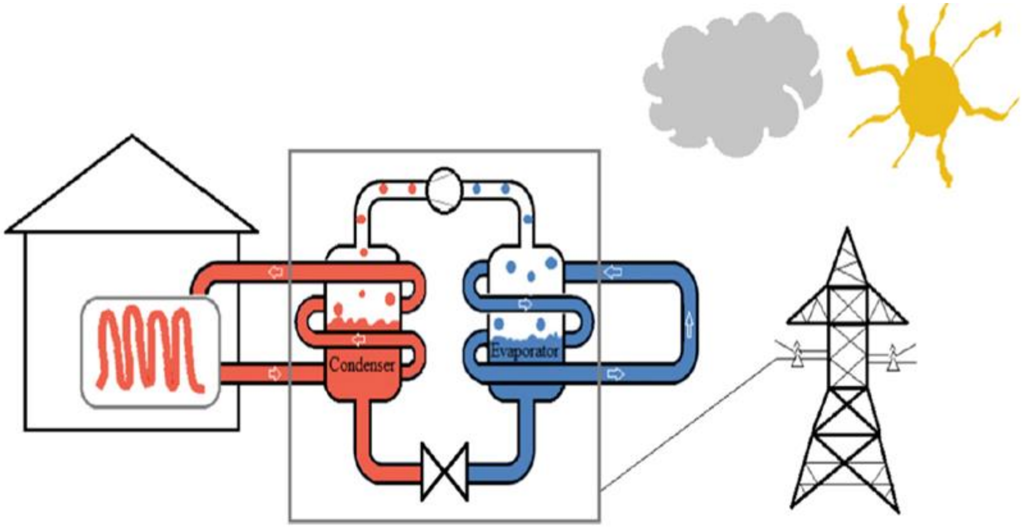
$$N = n_0 + C + P$$

Some consumers become prosumers

Augment the network with new arcs allowing prosumers to distribute energy to the network

Carroll, P., & Requejo, C. (2019, June). Smart Grid Topology Designs. In INOC 2019: 9th International Network Optimization Conference (INOC), Avignon, France, 12-14 June 2019 (pp. 49-53). OpenProceedings. org.

Heat Pump Efficiency Management



Carroll, P., Chesser, M., & Lyons, P. (2020). Air Source Heat Pumps field studies: A systematic literature review. *Renewable and Sustainable Energy Reviews*, *134*, 110275.

Green Vehicle Routing Problems

Data and Parameters

Travel Speed (km/hr), vehicle and goods weights, road network with gradients

Problem Statement

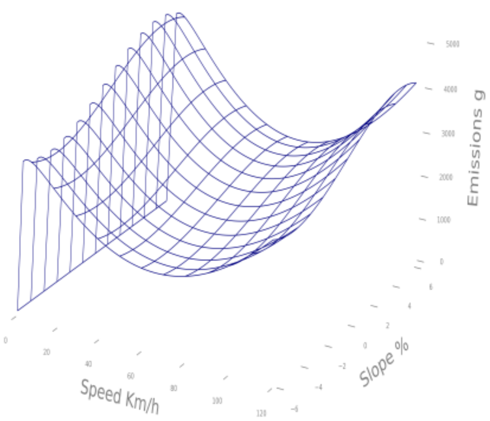
Determine a minimum emissions cost set of vehicle routes that satisfy capacity and driving constraints.

Xpreso were a startup software company for parcel delivery services; Practice was to solve an mTSP using third party solver software.

- Vehicle Category : Light duty vehicle
- Fuel Type : Diesel
- Weight Unloaded : $\leq$ 3.5 Tons
- Weight Loaded : $\leq$ 7.5 Tons

Green Vehicle Routing Problems

Gradient-Emissions Model



$$Emissions(kg\ of\ CO_2) = k + n * G + q * G^2 + r * V + \frac{u}{V}$$

$$V\left(\frac{Km}{h}\right) = Average\ Speed$$

$$G\ (\%) = Average\ Road\ Gradient$$

$$n, q, r, u = Coefficients$$

$$k = Constant = 1.27$$

Case Study in Bristol, UK

A software company aiming to provide optimization services to parcel delivery companies, particularly those with corporate social responsibility objectives.

da Costa, P. R. D. O., Mauceri, S., Carroll, P., & Pallonetto, F. (2018). A genetic algorithm for a green vehicle routing problem. *Electronic notes in discrete mathematics*, *64*, 65-74.

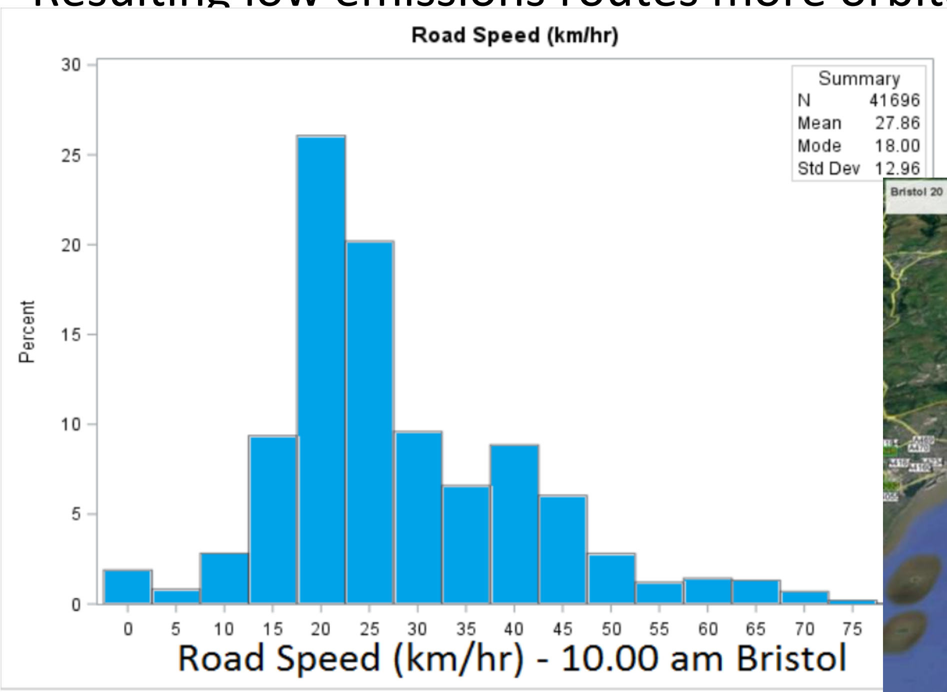
Green Vehicle Routing Problems

Bristol Case Study

Ex post routes data extracted via Google Maps API

Re-optimised using a Genetic Algorithm

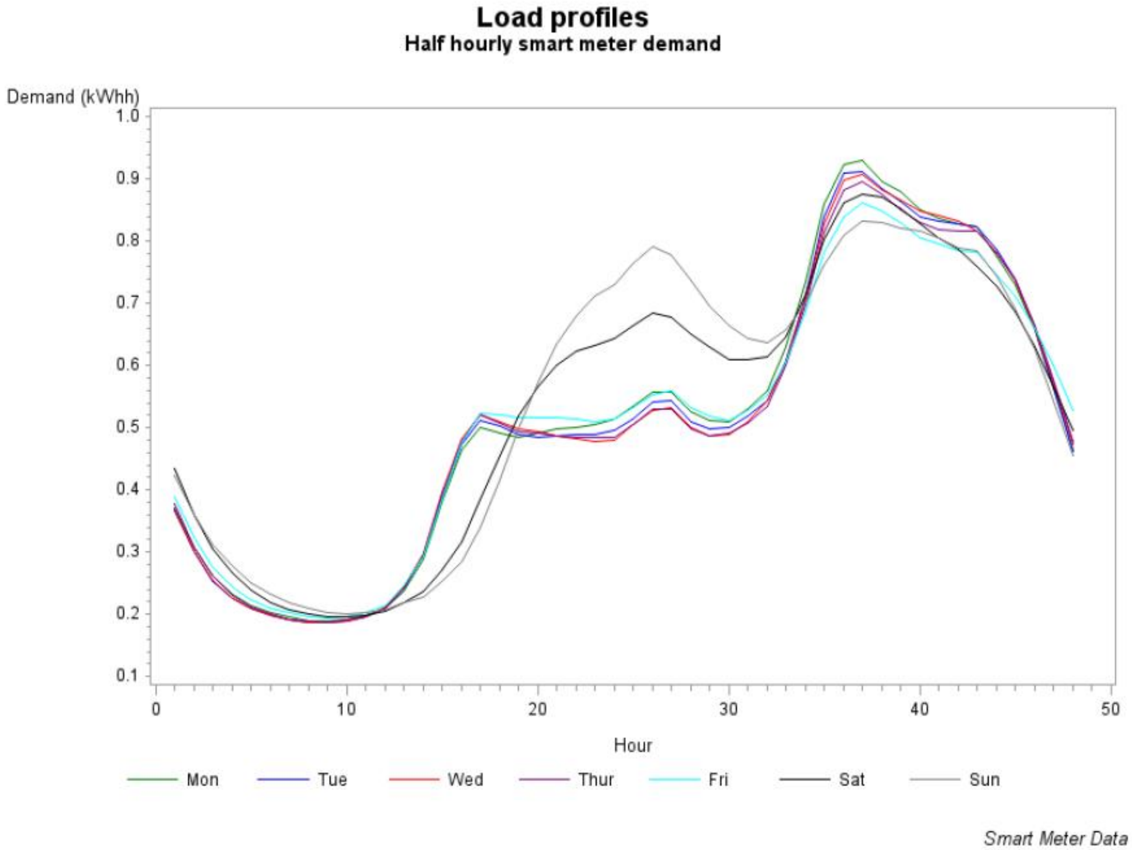
Resulting low emissions routes more orbital



Further work: Carroll, P., & Keenan, P. (2019). Decision Making Using Exact Optimization Methods in Sustainable Transportation. In *Sustainable Transportation and Smart Logistics* (pp. 263-283). Elsevier.

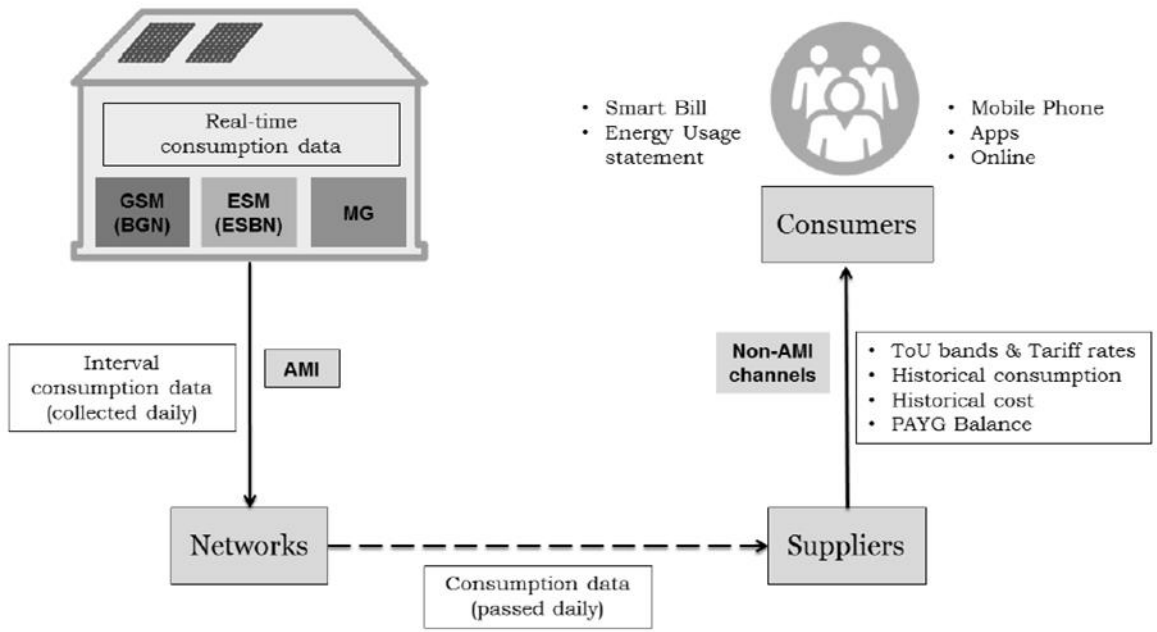
Electricity Smart Meter Data Exploration

The Irish Commissioner for Energy Regulation ran a consumer behaviour smart meter trial in 2009.
The data are available to evaluate load classification/clustering/characterising



Electricity Smart Meter Data Exploration

Carroll, P., Murphy, T., Hanley, M., Dempsey, D., & Dunne, J. (2018). Household Classification using Smart Meter Data. *Journal of Official Statistics*, *34*(1), 1-25.



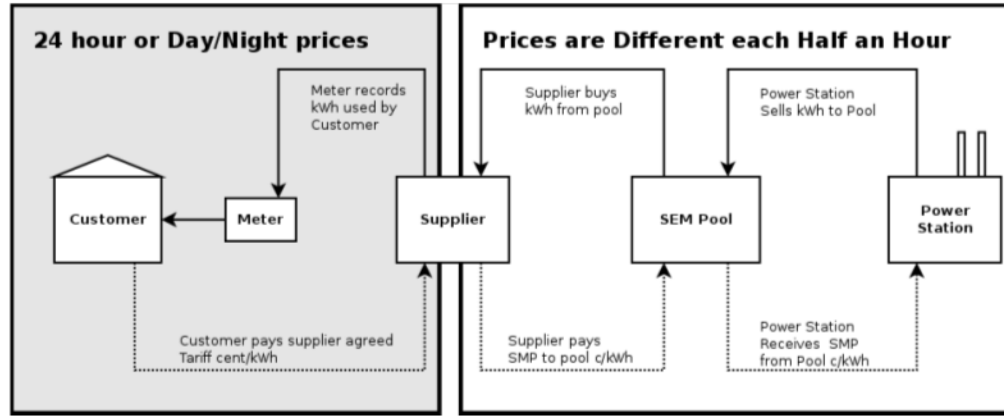
GSM – Gas Smart Meter provided by Bord Gais Networks (BGN)
ESM – Electricity Smart Meter provided by Electricity Supply Board Networks (ESBN)
MG – Micro-Generation meter provided by parties yet to be determined
AMI – Automated Meter Infrastructure
ToU – Time of Use
PAYG – Pay As You Go (enhanced form of PrePayment)

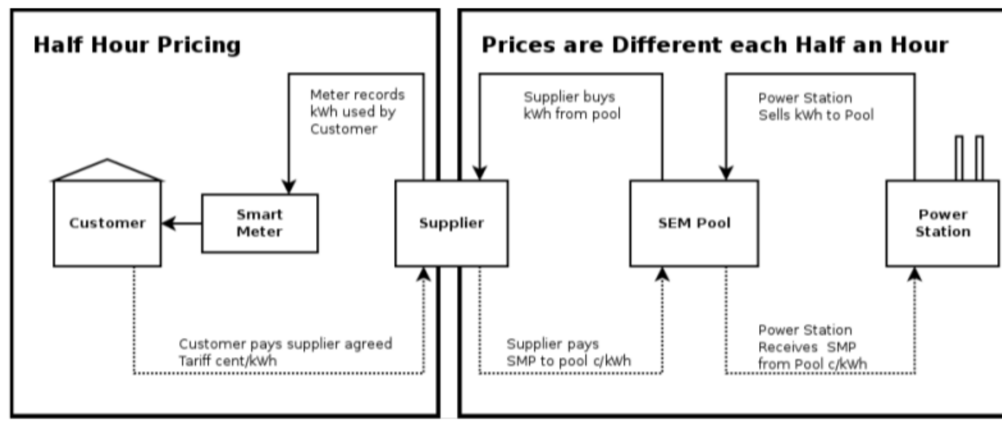
https://www.cru.ie/wp-content/uploads/2014/07/CER14046-High-Level-Design.pdf

Electricity Smart Meter Data Exploration

Rogers, W., Carroll, P., & McDermott, J. (2019). A Genetic Algorithm Approach to the Smart Grid Tariff design problem. *Soft Computing*, 23(4), 1393-1405.

Carroll, P. (2020, July). Exploring Smart Grid Time-of-Use Tariffs using a Robust Optimisation Framework. In 2020 *International Joint Conference on Neural Networks (IJCNN)* (pp. 1-6). IEEE.





Summary

Observations

- Higher renewable energy availability, yield more challenging MILP UC problem instances:
- The electricity market solves large "day-ahead" MILP models – the integrality gap is closed in real time to match the outturn demand with electricity supply and demand response products;
- Smart Grid mesh structures are more resilient & more supportive of energy community objectives, but present more management challenges – the telecommunications network design and management expertise of the OR community is transferable
- **Lots of Optimisation and Data Science Problems to solve to enable the transformation of the energy system!!!**

Future Work

Research Directions

- Physical Infrastructure versus Logical (Virtual) community network design - Existing Grid – Network Mapping;
- More detailed RES and Demand models;
- Microgrid Design – locating and then matching supply with demand nodes;
- Low Carbon Technology Load Modelling;
- GVRP and EV problem instances and algorithms;
- Addressing Fairness concerns.

Connections

The EURO WISDOM Forum aims to provide a platform to support, empower and encourage the participation of all genders in Operational Research within EURO
https://www.euro-online.org/web/pages/1654/wisdom

UCD DUBLIN UCD College of Business

IORMS Institute of Operational Research and Management Science of Nigeria

Thanks for listening!

Questions?

Paula Carroll, PhD

WISDOM

WOMEN IN SOCIETY:
DOING OPERATIONAL RESEARCH
AND MANAGEMENT SCIENCE

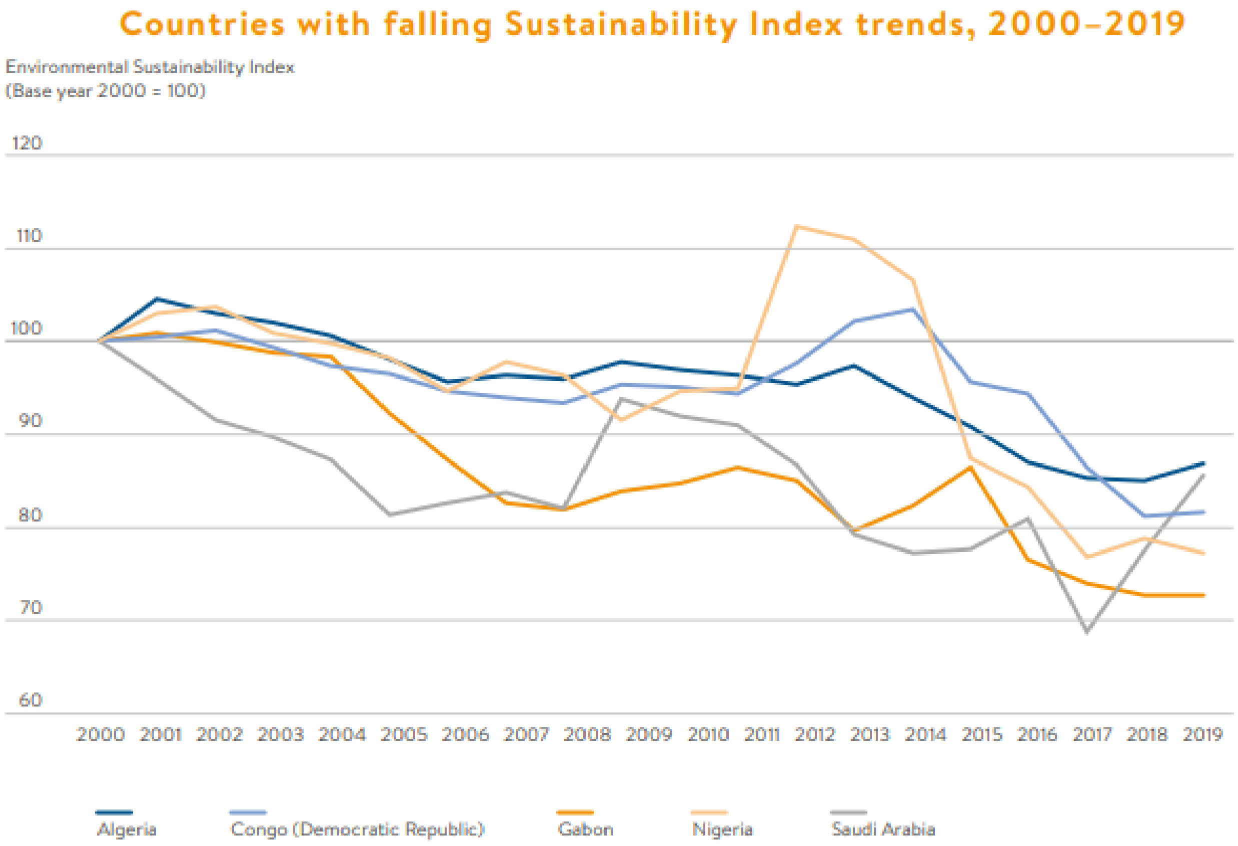
UCD ENERGY INSTITUTE

Data Sources

Electricity & Weather Data

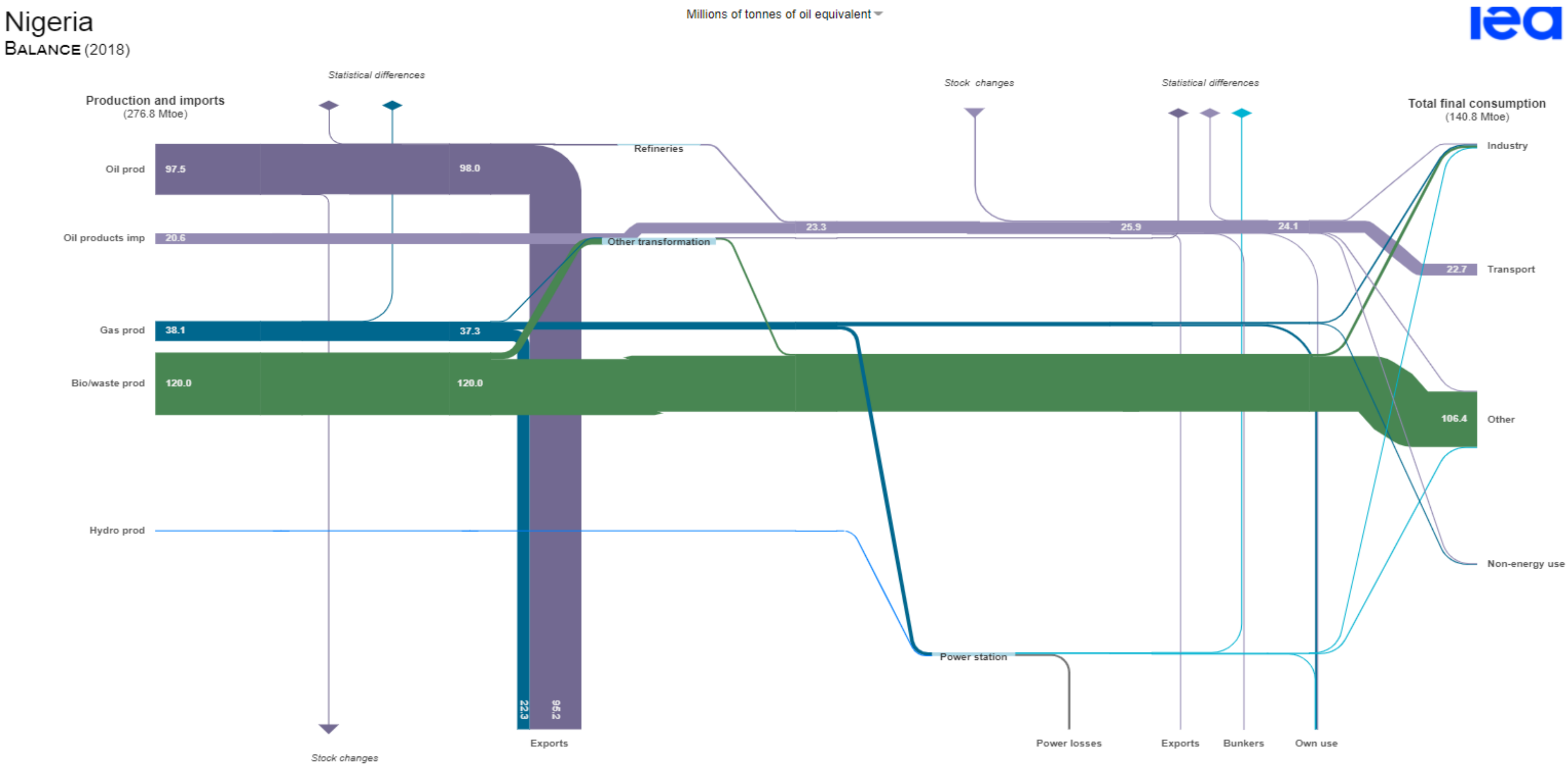
- Irish Smart Meter Trial: http://www.ucd.ie/issda/data/commissionforenergyregulationcer/
- http://smartgriddashboard.eirgrid.com/
- http://meteireann.ie

Decreasing Sustainability Index



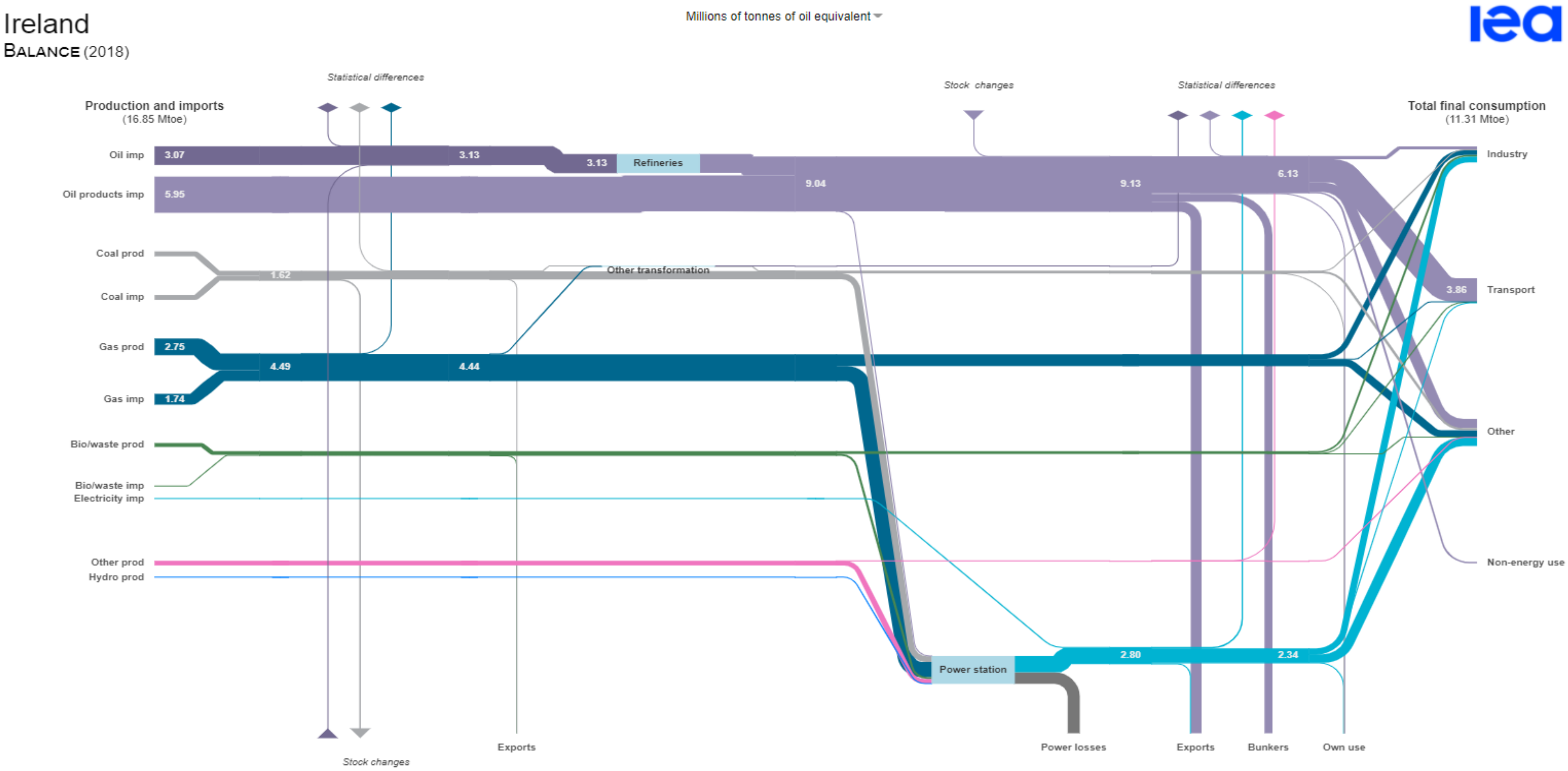
Source: World Energy Trilemma Index 2019

Nigeria's Energy Balance 2018



Source: https://www.iea.org/sankey

Ireland's Energy Balance 2018

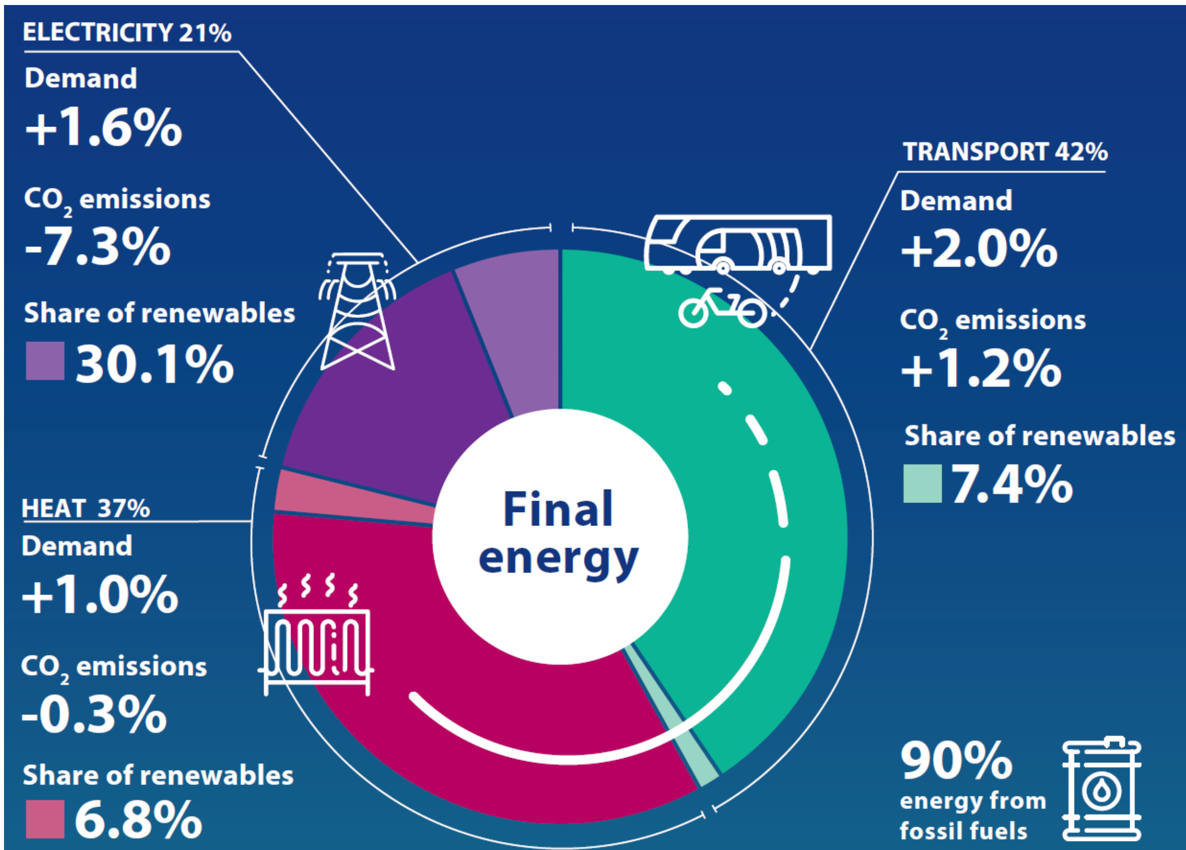


Source: https://www.iea.org/sankey

UCD ENERGY INSTITUTE

UCD College of Business

Ireland's Energy Requirements



SEAI, Energy in Ireland, 2018 Report.
https://www.seai.ie/publications/Energy-in-Ireland-2018.pdf

Discipline Boundaries

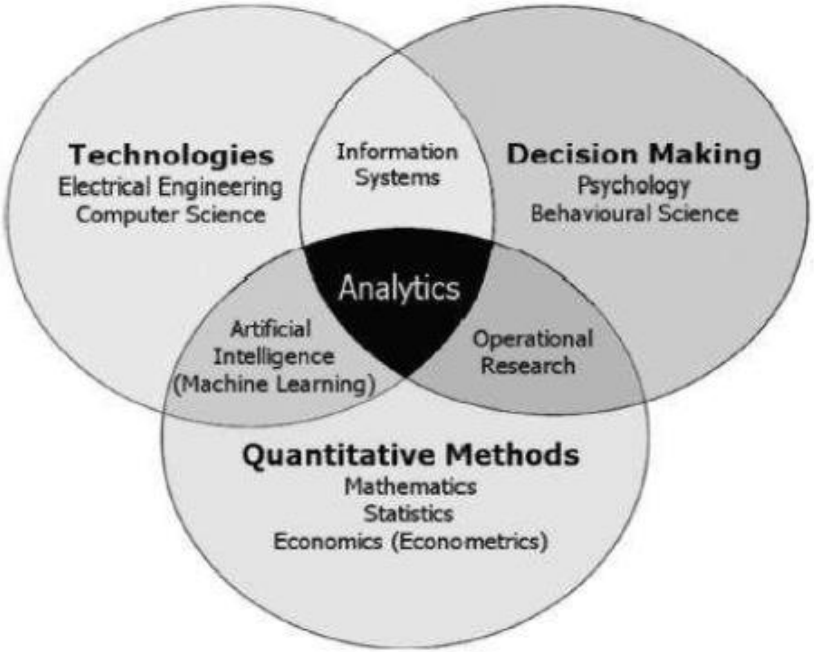


Figure 1: Business Analytics Disciplinary Positioning, Mortenson et al. (2015)

Mortenson, M. J., Doherty, N. F., & Robinson, S. (2015). Operational research from Taylorism to Terabytes: A research agenda for the analytics age. *European Journal of Operational Research*, 241(3), 583-595.

Methodology – GVRP B&C Algorithm

- Vehicle Flow formulation[1] with some heuristics at the root node
- XpressMP 8.5 on a Dell 64 bit Windows 8 machine with Intel i5 3.2 GHz processor and 8GB of Ram

(VF) $$Min \sum_{ij \in E} c_{ij} x_{ij} \quad (1)$$

Subject to:

$$\sum_{j \in V \setminus \{1\}} x_{1j} = 2m \quad (2)$$

$$\sum_{i \in V \mid i<j} x_{ij} + \sum_{k \in V \mid j<k} x_{jk} = 2 \quad \forall j \in V \setminus \{1\} \quad (3)$$

$$\sum_{i,j \in S} x_{ij} \leq |S| - V(S) \quad \forall S \subseteq V \setminus \{1\}, \ |S| \geq 2 \quad (4)$$

$$x_{1j} \leq 2 \quad \forall j \in V \setminus \{1\} | c_{1j} \leq L/2 \quad (5)$$

$$x_{ij} \leq 1 \quad \forall i,j \in V \setminus \{1\} | \ i < j \quad (6)$$

$$x_{ij} \in \mathbb{Z}^+ \quad \forall i,j \in V \setminus \{1\} | \ i < j \quad (7)$$

- x_{ij} are binary decision variable, c_{ij} are emissions costs

1 Laporte, Gilbert, Yves Nobert, Martin Desrochers. 1985. Optimal routing under capacity and distance restrictions. *Operations Research* **33** 1050-1073.

Carroll, P., & Keenan, P. (2019). Decision Making Using Exact Optimization Methods in Sustainable Transportation. In *Sustainable Transportation and Smart Logistics* (pp. 263-283). Elsevier.

Enumerative Numerical Solution for Optimal Control using Treatment and Vaccination for an SIS Epidemic Model

IORMS VIRTUAL CONFERENCE 2020

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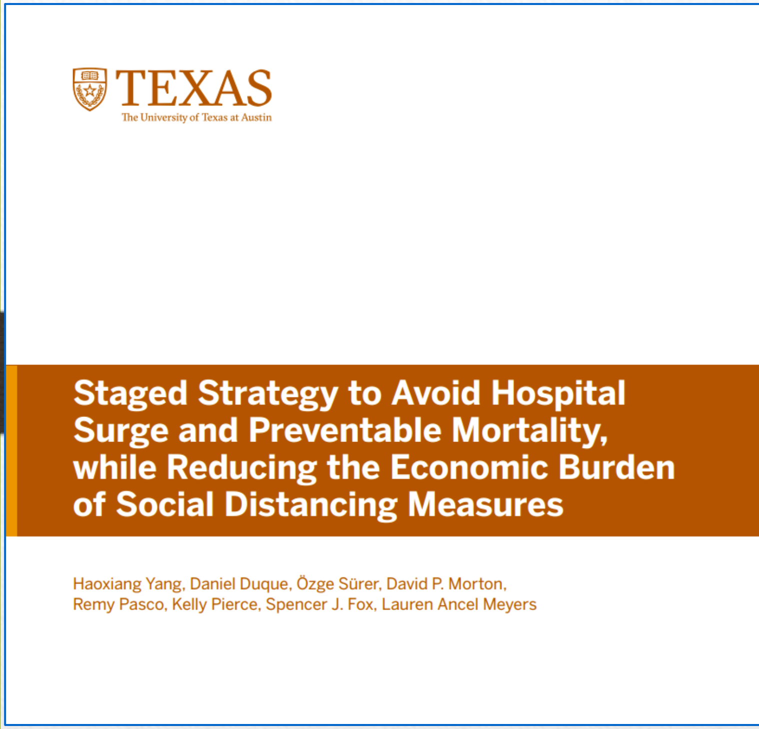
Vianney Mbazumutima
Institute of Mathematics and Physical Sciences
Porto Novo, Benin

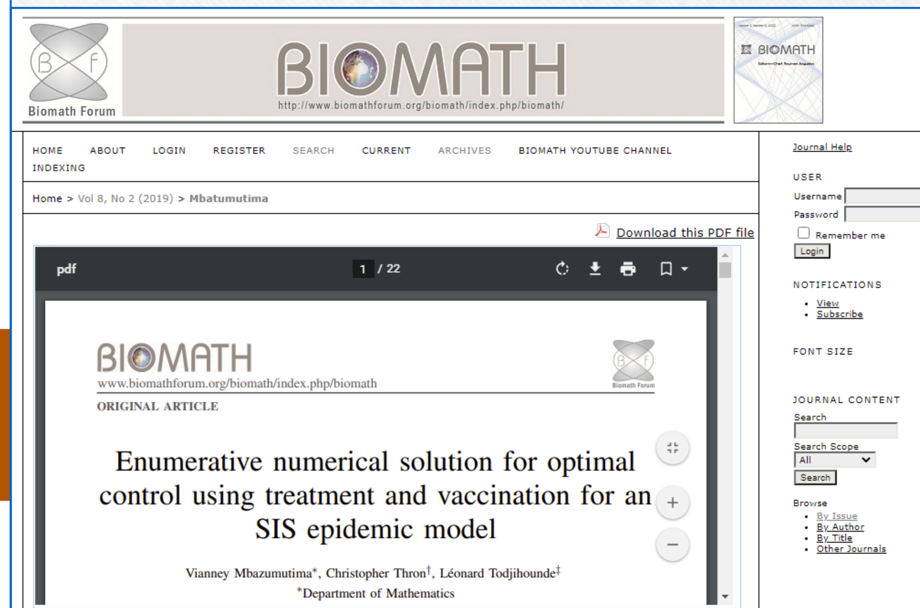
This presentation is based on the following published references:



TEXAS
The University of Texas at Austin

Staged Strategy to Avoid Hospital Surge and Preventable Mortality, while Reducing the Economic Burden of Social Distancing Measures

Haoxiang Yang, Daniel Duque, Özge Sürer, David P. Morton, Remy Pasco, Kelly Pierce, Spencer J. Fox, Lauren Ancel Meyers



BIOMATH
www.biomathforum.org/biomath/index.php/biomath
ORIGINAL ARTICLE

Enumerative numerical solution for optimal control using treatment and vaccination for an SIS epidemic model

Vianney Mbazumutima*, Christopher Thron†, Léonard Todjihounde‡
*Department of Mathematics

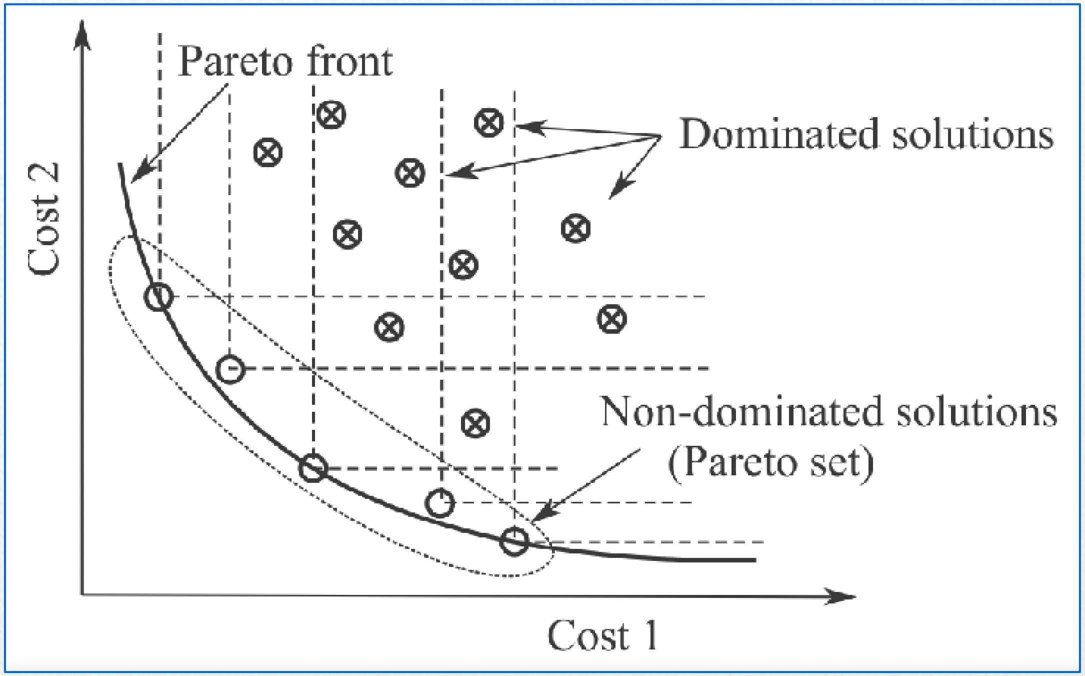
https://sites.cns.utexas.edu/sites/default/files/cid/files/houston_strategy_to_avoid_surge.pdf?m=1592489259

http://www.biomathforum.org/biomath/index.php/biomath/article/view/j.biomath.2019.12.137

Presentation Outline

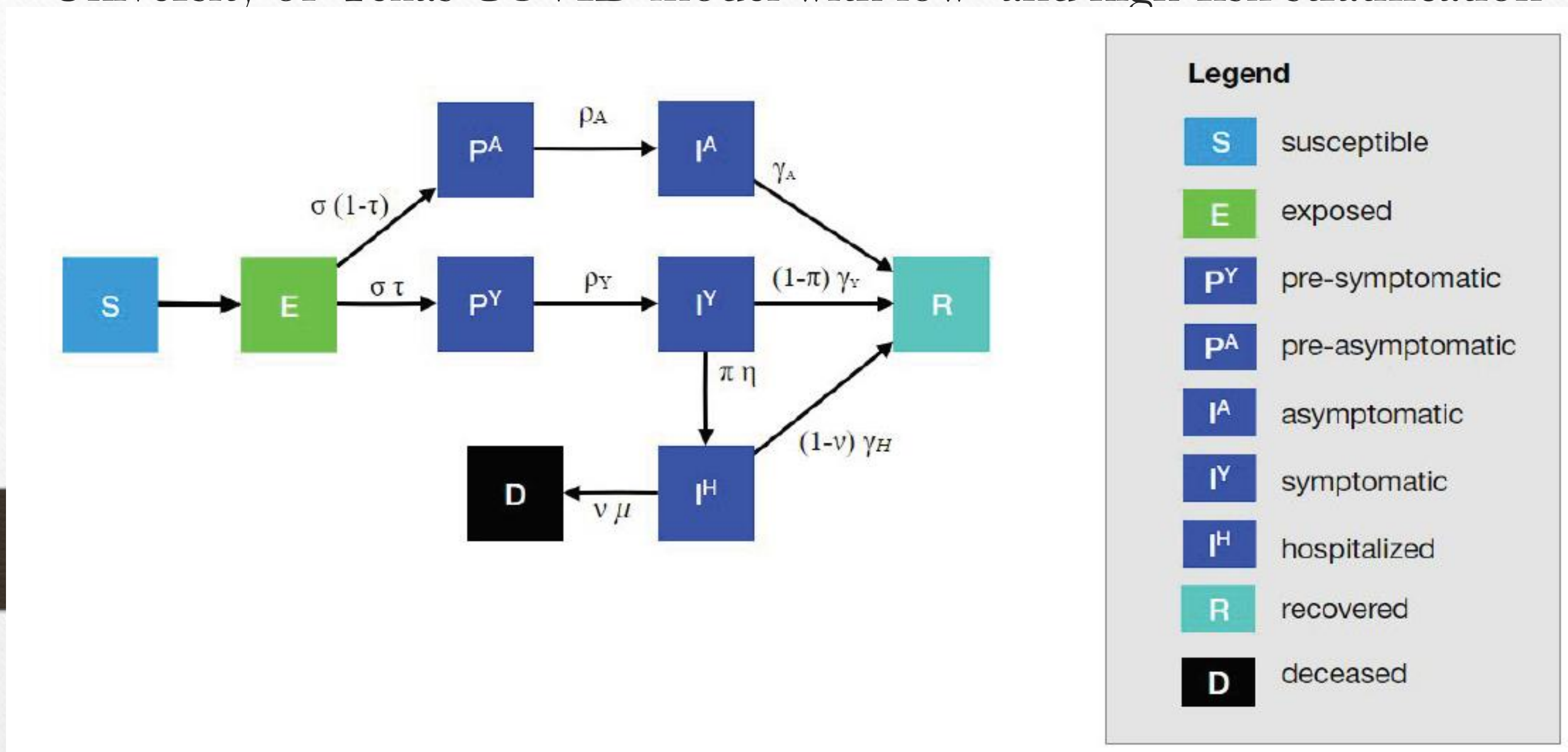
- What is meant by "optimal"?
- Optimizing implementation cost:
  - Example: COVID model with testing and distancing
- Optimizing overall cost:
  - Factors to consider
  - Global versus local optimization
- Approaches to global optimization:
  - Solution parametrization
  - Enumerative solution
  - Local refinement
- Lessons learned

What is meant by "optimal"?



- Any action, policy, or strategy has consequences that are either beneficial or detrimental.
- Typically, one would like to chose the policy that yields the overall greatest benefit.
- It may be hard to compare the relative benefits of different consequences.
- One approach is to attach a monetary value to different consequences, to create a real-valued *cost function*
- Another approach is to identify a set of *non-dominated solutions* that cannot be improved upon (Pareto optimal front)
- In this paper, we use a cost function.

University of Texas COVID model with low- and high-risk stratification



- Susceptible: not yet infected
- Exposed: infected, not yet infectious
- Presymptomatic: infectious, not symptomatic
- Infectious: may be symptomatic or asymptomatic
- Hospitalized, death: serious cases
- Recovered: assumed to be immune

Duplicate graph for low- and high-risk. 18 compartments total.

Basic & Effective Reproduction numbers

The **basic reproduction number (R_0)** is defined as the average number of additional infections produced by a single infection, assuming that everyone is susceptible.

The **effective reproduction number (R_e)** is defined as the average number of additional infections produced by a single infection in the current mixed population.

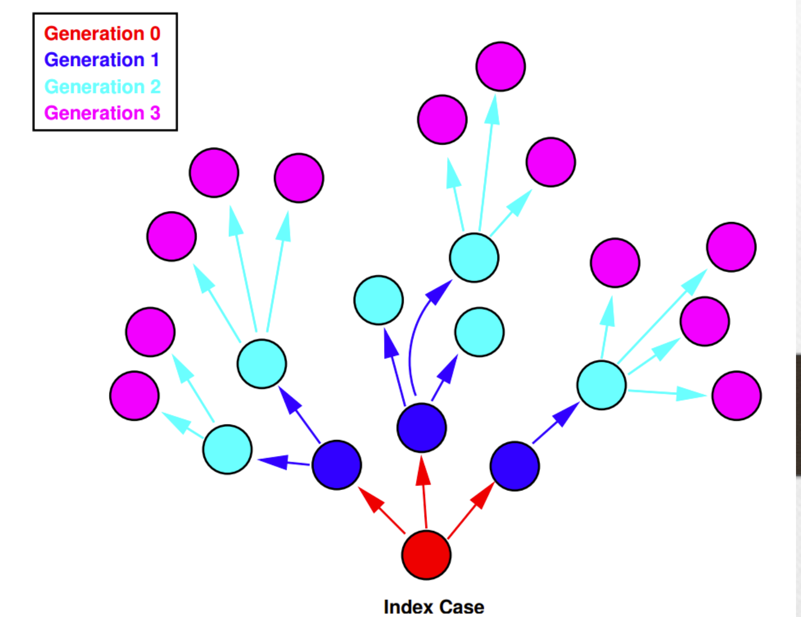


Figure 2: Graphical depiction of "generations" in an epidemic.

One index case has produced 12 3rd generation cases. The reproduction number is approximately $(12)^{1/3} \approx 2.2$.

https://web.stanford.edu/~jhj1/teachingdocs/Jones-on-R0.pdf

Mathematical formulation of COVID-19 epidemic model under controls

$$\frac{dS_j}{dt} = -\sum_{i=0}^{1} \frac{1}{N_i} \left(I_i^Y \omega^Y + (1 - u_i)[I_i^A \omega^A + P_i^Y \omega^{PY} + P_i^A \omega^{PA}] \right) (1 - v_j) \beta \Phi_{ji} S_j,$$

$$\frac{dE_j}{dt} = \sum_{i=0}^{1} \frac{1}{N_i} \left(I_i^Y \omega^Y + (1 - u_i)[I_i^A \omega^A + P_i^Y \omega^{PY} + P_i^A \omega^{PA}] \right) (1 - v_j) \beta \Phi_{ji} S_j - \sigma E_j.$$

$$\frac{dP_j^A}{dt} = (1 - \tau)\sigma E_j - \rho^A P_j^A$$

$$\frac{dP_j^Y}{dt} = \tau \sigma E_j - \rho^Y P_j^Y$$

$$\frac{dI_j^A}{dt} = \rho^A P_j^A - \gamma^A I_j^A$$

$$\frac{dI_j^Y}{dt} = \rho^Y P_j^Y - (1 - \Pi_j)\gamma^Y I_j^Y - \Pi_j \eta I_j^Y$$

$$\frac{dI_j^H}{dt} = \Pi_j \eta I_j^Y - (1 - \nu_j)\gamma^H I_j^H - \mu \nu_j I_j^H$$

$$\frac{dR_j}{dt} = \gamma^A I_j^A + (1 - \Pi_j)\gamma^Y I_j^Y + (1 - \nu_j)\gamma^H I_j^H$$

$$\frac{dD_j}{dt} = \mu \nu_j I_j^H.$$

u_i = Testing level for $i'th$ population group

v_i = Distancing level for $i'th$ population group

i= 0,1 (low risk,high risk)

Model was implemented in Python

(Anaconda /Spyder / numpy)

Implementation cost model for testing and distancing

$$\alpha_j(u_j, N_j^A) = \begin{cases} 0 & \text{if } \; u_j = 0, \\ a_{j0} + a_{j1} N_j^A u_j + a_{j2} N_j^A (u_j)^2 & \text{if } \; 0 < u_j \le u_j^{(max)}, \end{cases}$$

$$\beta_j(v_j) = b_{j1} N_j v_j + b_{j2} N_j (v_j)^n \qquad \text{if } \; 0 < v_j \le v_j^{(max)}.$$

$$N_j^A = S_j + E_j + P_j^A + P_j^Y + I_j^A$$

no. asymptomatic individuals in subpopulation.

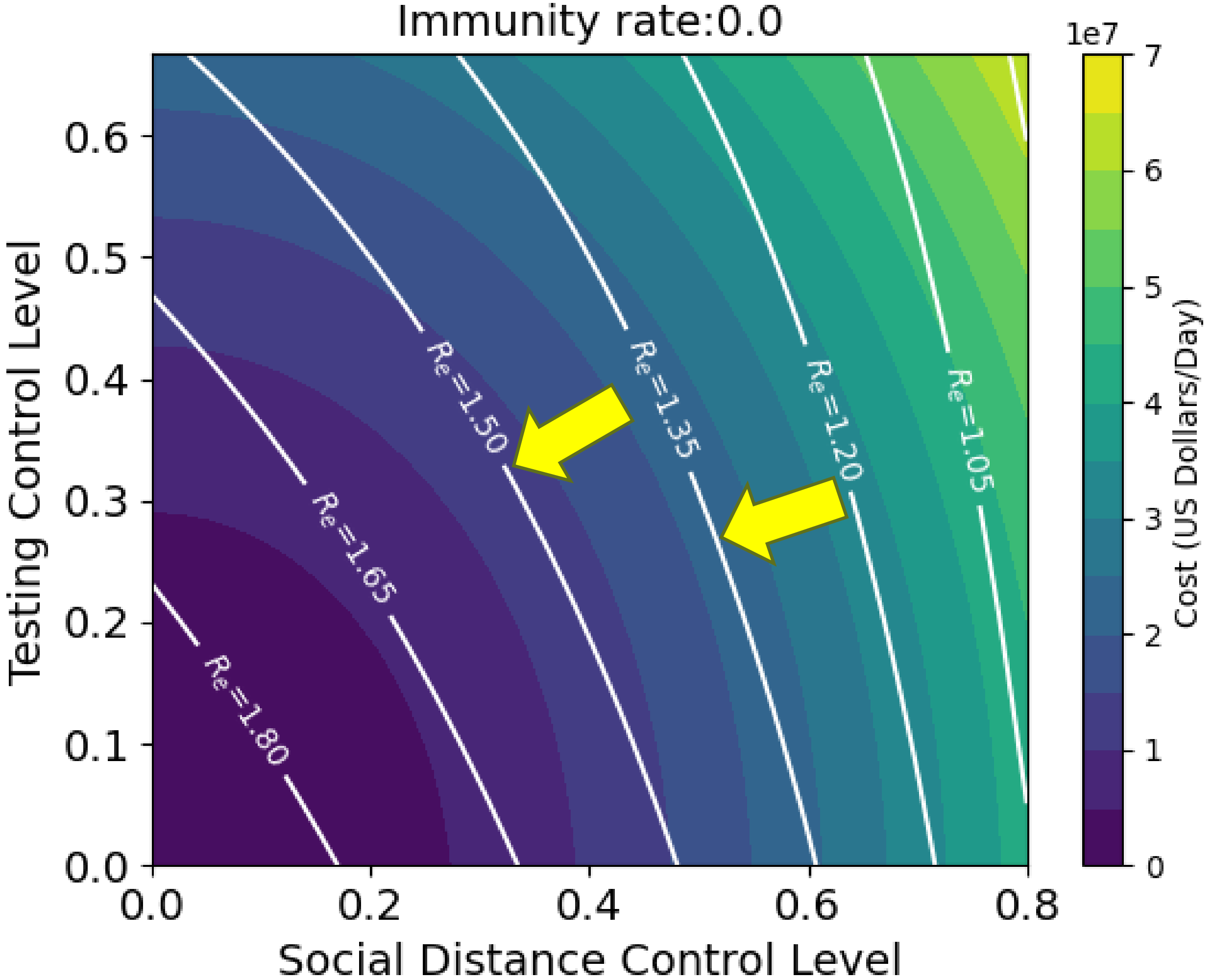
α_j and β_j = costs for COVID testing and social distancing respectively for subpopulations j=0,1 (low risk, high risk)

Nonlinear costs reflect the *law of diminishing returns* (higher level of controls have higher marginal costs)

List of model parameters

Parameters	Interpretation	Values
β	baseline transmission rate	0.0640
γ^A	recovery rate on asymptomatic compartment	Equal to γ^Y
γ^Y	recovery rate on symptomatic non-treated compartment	$\frac{1}{\gamma^Y} = 4.0$
τ	symptomatic proportion	0.55
σ	exposed rate	$\frac{1}{\sigma} \sim 2.9$
ρ^A	pre-asymptomatic rate	Equal to ρ^Y
ρ^Y	pre-symptomatic rate	$\frac{1}{\rho^Y} = 2.3$
P	proportion of pre-symptomatic transmission	0.44
ω^Y	relative infectiousness of symptomatic individuals	1.0
ω^P	relative infectiousness of pre-symptomatic individuals	$\omega^P = \frac{P}{1-P}\frac{\tau\omega^Y[YHR/\eta+(1-YHR)/\gamma^Y]+(1-\tau)\omega^A/\gamma^A}{\tau\omega^Y/\rho^Y+(1-\tau)\omega^A/\rho^A}, \omega^{PY} = \omega^P\omega^Y, \omega^{PA} = \omega^P\omega^A$
ω^A	relative infectiousness of infectious individuals in compartment I^A	$\omega_A = 0.66$
IFR	infected fatality ratio, age specific (%)	$[0.6440, 6.440]$
YFR	symptomatic fatality ratio, age specific (%)	$[1.130, 11.30]$
γ^H	recovery rate in hospitalized compartment	$\frac{1}{\gamma^H} \sim 10.7$
YHR	Symptomatic case hospitalization rate %	$[4.879, 48.79]$
Π	rate of symptomatic individuals go to hospital, age-specific	$\Pi = \frac{\gamma^Y * YHR}{\eta+(\gamma^Y-\eta)YHR}$
η	rate from symptom onset to hospitalized	0.1695
μ	rate at which terminal patients die	$\frac{1}{\mu} = 8.1$
HFR	hospitalized fatality ratio, age specific (%)	$[4, 23.158]$
ν	death rate on hospitalized individuals, age specific	$\nu = \frac{\gamma^H HFR}{\mu+(\gamma^H-\mu)HFR}$

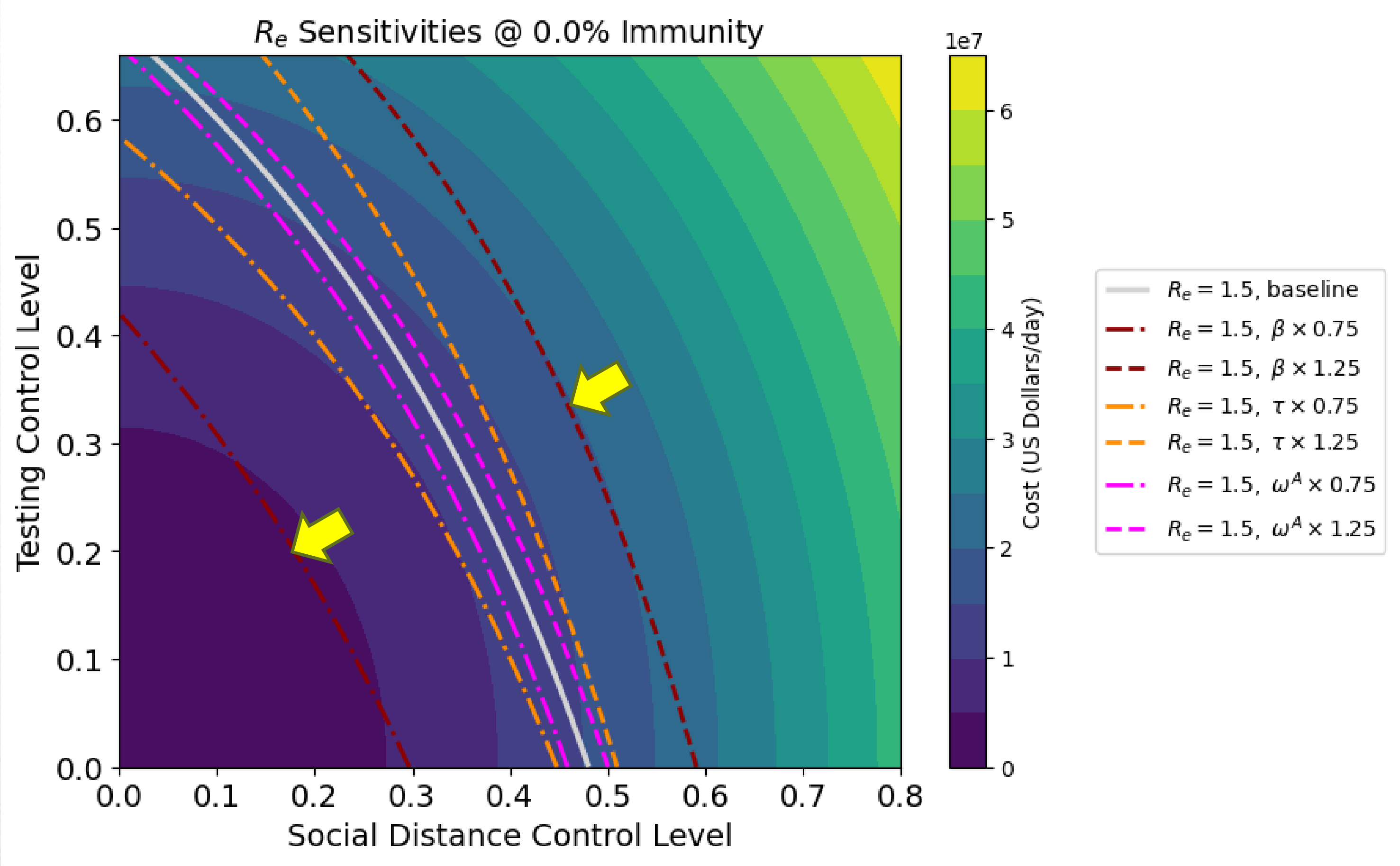
Testing vs. distancing costs (assuming population-wide implementations)



Similar tradeoffs between cost and effective reproduction number at different levels of immunity

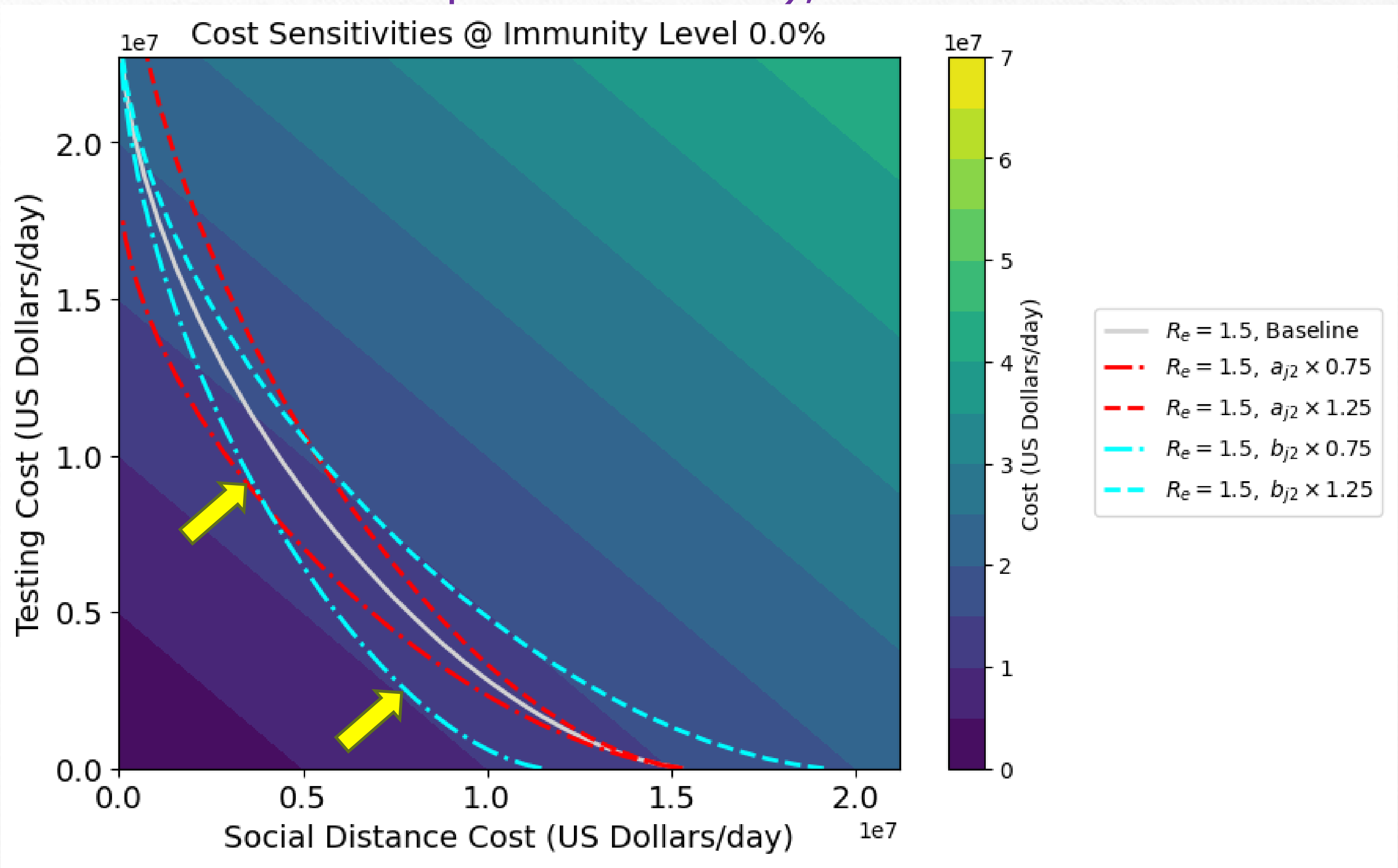
(Graphs were made with countour() function from matplotlib.pyplot)

Testing vs. distancing costs (assuming population-wide implementations), ctd.



- Cost is highly sensitive to model parameter values
- Tradeoff between testing and distancing is much less sensitive

Testing vs. distancing costs (assuming population-wide implementations), ctd.

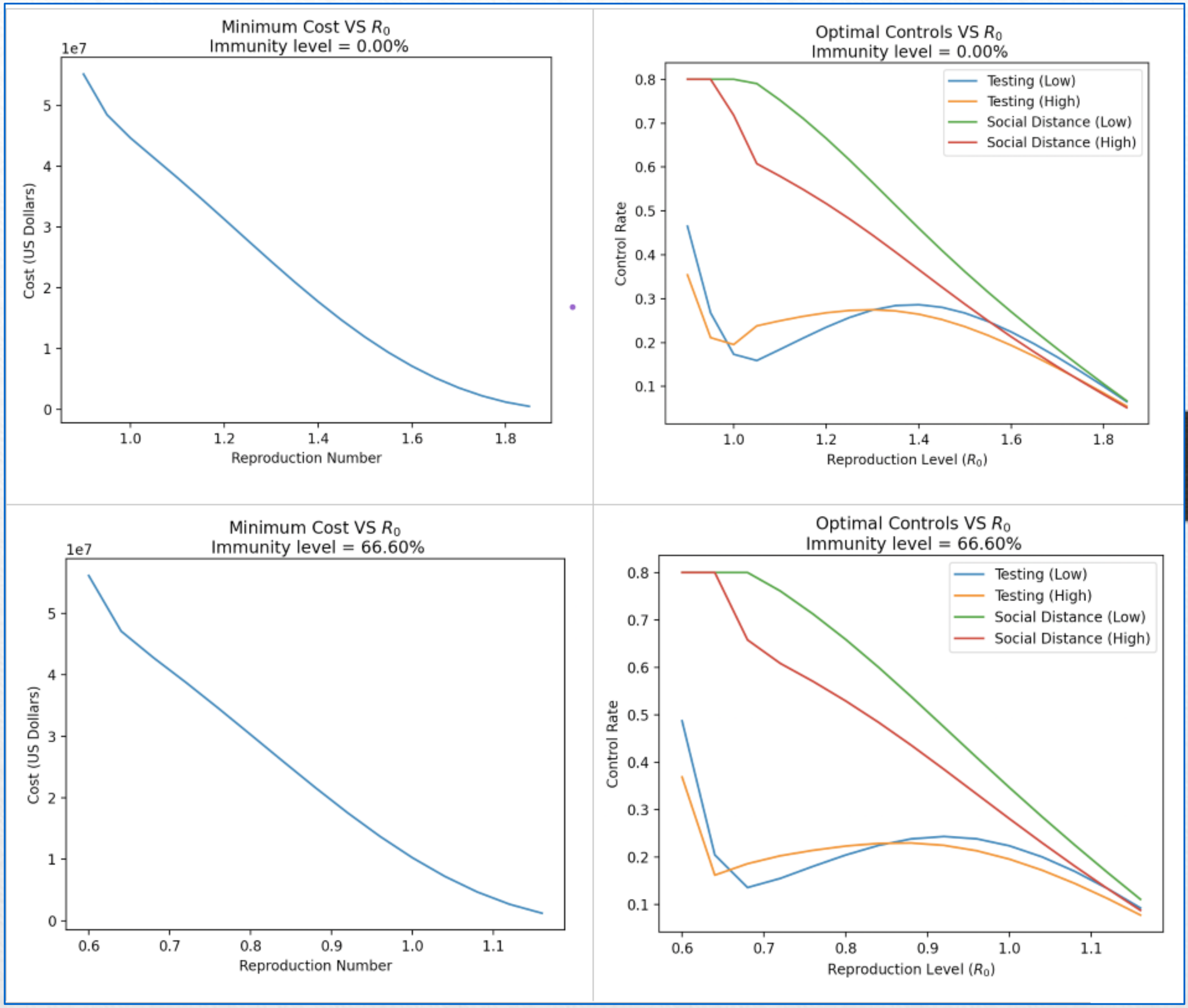


- Optimal tradeoff depends on cost model

Detailed tradeoff between testing and distancing

Lowest-cost mix of 4 control strategies for given reproduction number is computed using scipy.optimize

Surprisingly, the solution scales for different immunity levels.



Optimization of overall costs

Besides optimization cost, there are several other factors that have monetary or quality-of-life costs:

$$J_{tot}(\overline{\mathbf{u}}, \overline{\mathbb{X}}) = \int_0^{t_f} \sum_{j=0}^{1} J_j(u_j, v_j, \mathbb{X}_j)dt + \sum_{j=0}^{1} \left(e_j D_j(t_f) + f_j I_j^A(t_f)\right),$$

$$J_j(u_j, v_j, \mathbb{X}_j) = \alpha_j(u_j, N_j^A) + \beta_j(v_j) + c_j I_j^Y + d_j I_j^H \quad (j = 0, 1),$$

$$\alpha_j(u_j, N_j^A) = \begin{cases} 0 & \text{if } \; u_j = 0, \\ a_{j0} + a_{j1} N_j^A u_j + a_{j2}(N_j^A u_j)^2 & \text{if } \; 0 < u_j \le u_j^{(max)}, \end{cases}$$

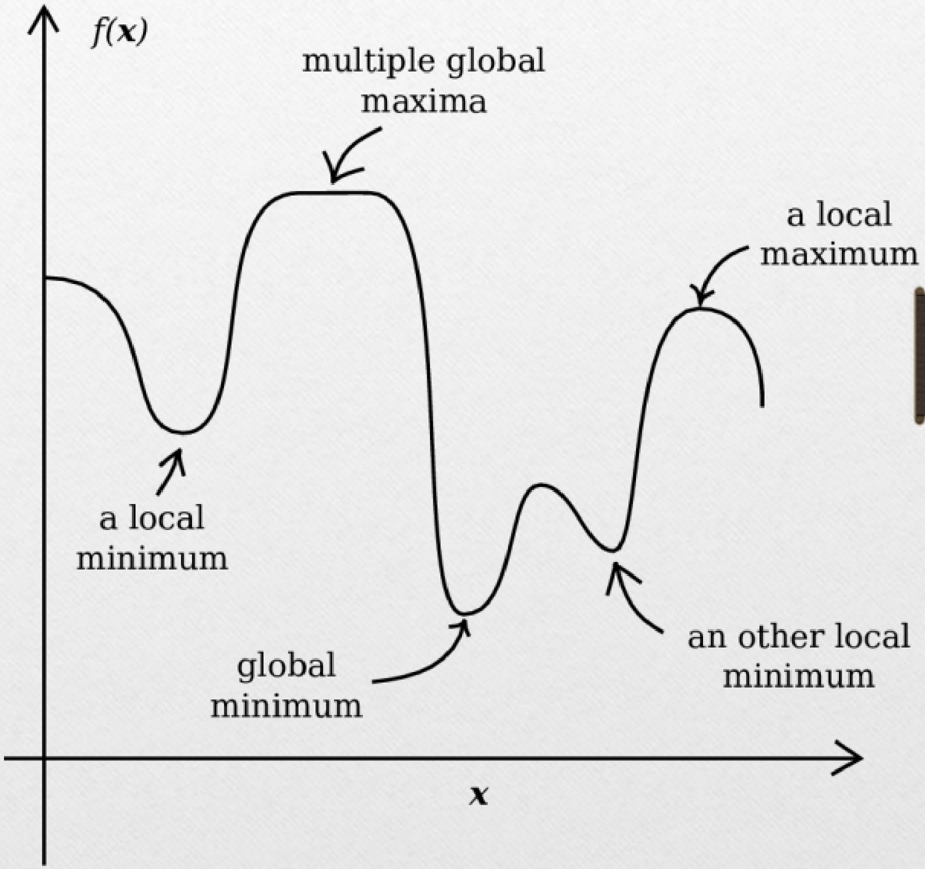
$$\beta_j(v_j) = b_{j0} + b_{j1} v_j + b_{j2} v_j^2 \qquad \text{if } \; 0 < v_j \le v_j^{(max)}.$$

c_j: Sickness cost per day
d_j: Hospitalization cost per day
e_j: Cost per death
f_j: Cost per residual infectious individual at final time t_f

j=0,1 for low risk / high risk

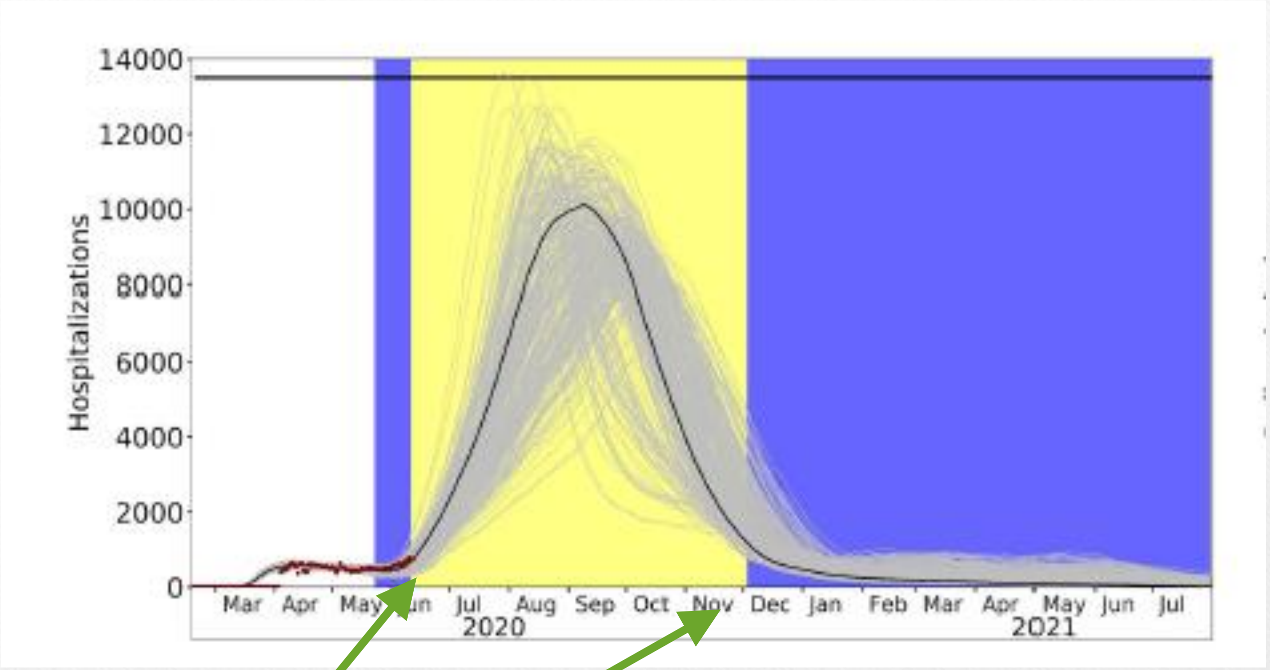
Global vs. local optimization

- Local optimization is much easier than global optimization.
- For some special cost functions, local optimization implies global optimization (Arrow's theorem).
- Most realistic control functions do not satisfy the conditions for Arrow's theorem.
- For this reason, *heuristic* or *stochastic methods* are typically used to find global optima.
- These methods are *inexact*, and can produce a good (but not necessarily the best) solution.



https://ludovicarnold.com/teaching/optimization/problem-statement/

Parametrized optimization



Consider only strategies with:

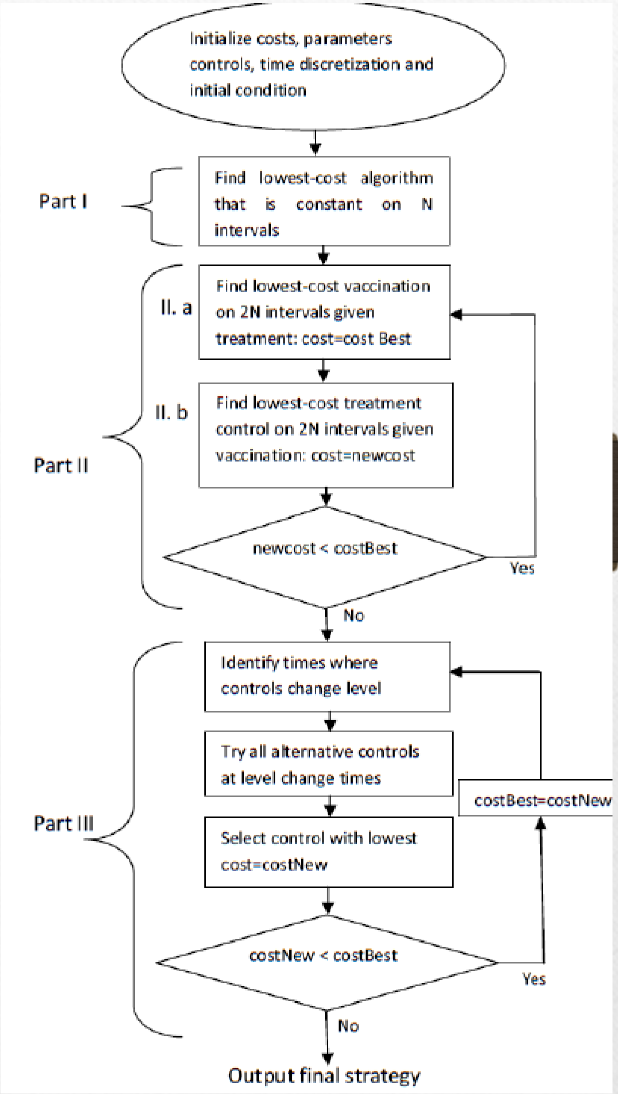
- fixed start time t_{start}
- fixed stop time t_{stop}
- fixed control levels u_0, u_1, v_0, v_1 (lo / hi risk testing distancing)
- The scipy.optimize function in python can be used to minimize the cost as a function of 6 parameters.

https://ludovicarnold.com/teaching/optimization/problem-statement/

Enumerative optimization (I)

Discretize possible solutions by:

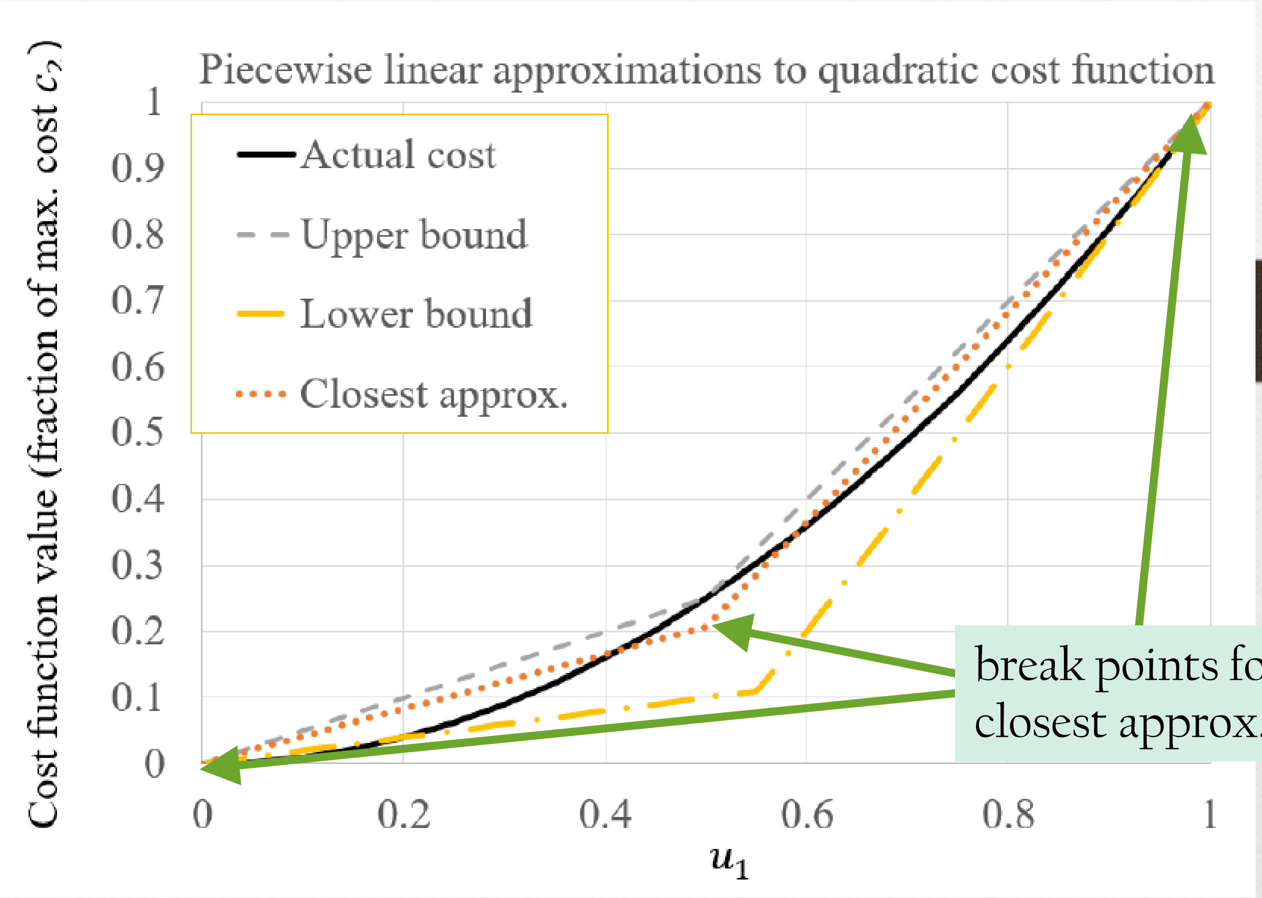
- Discretizing possible control levels and intervals of constant control
- (Part I) Evaluate costs for all controls in the discretized set
- (Part II) Subdivide intervals and hold some controls fixed while varying others. Evaluate costs, and choose the best solution.
- Repeat Part II until no more improvement is observed.
- (Part III) Adjust interval lengths for active intervals so as to reduce cost. Iterate until no more improvement is observed



Enumerative optimization (II): discretize control levels

Discretizing control levels corresponds to replacing continuous cost function with piecewise linear function

(For piecewise linear cost functions, it may be shown that the optimal control is always at a break point)



Enumerative optimization (III): Set strategy on fixed intervals

Part I: Initial intervals, 4^N strategies tried

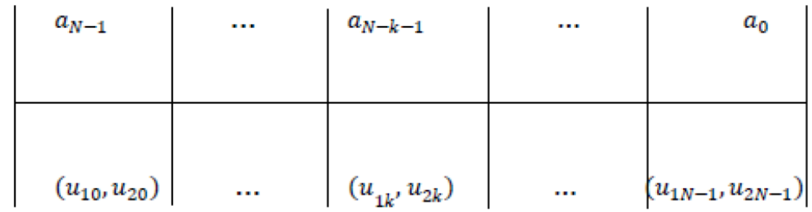


Figure 2: Strategies computed on k^{th} interval.

Part IIa: Subdivide intervals, 4^N strategies tried

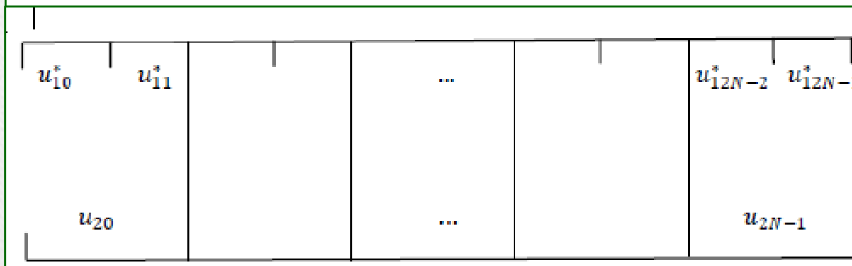


Figure 4: Schema of flowchart part II.a.

Part IIb: Subdivide intervals, 4^N strategies tried

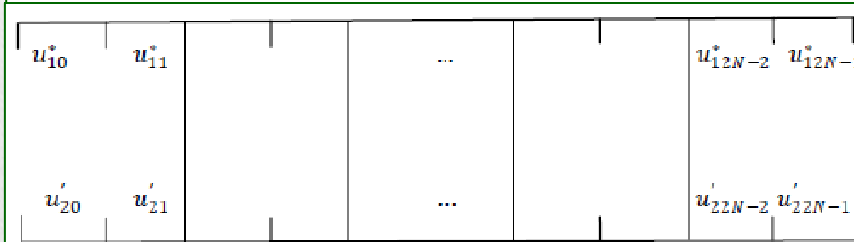


Figure 5: Schema of flowchart part II.b.

Enumerative optimization (IV): fine tuning of intervals

Part III: Interval edges are incrementally changed

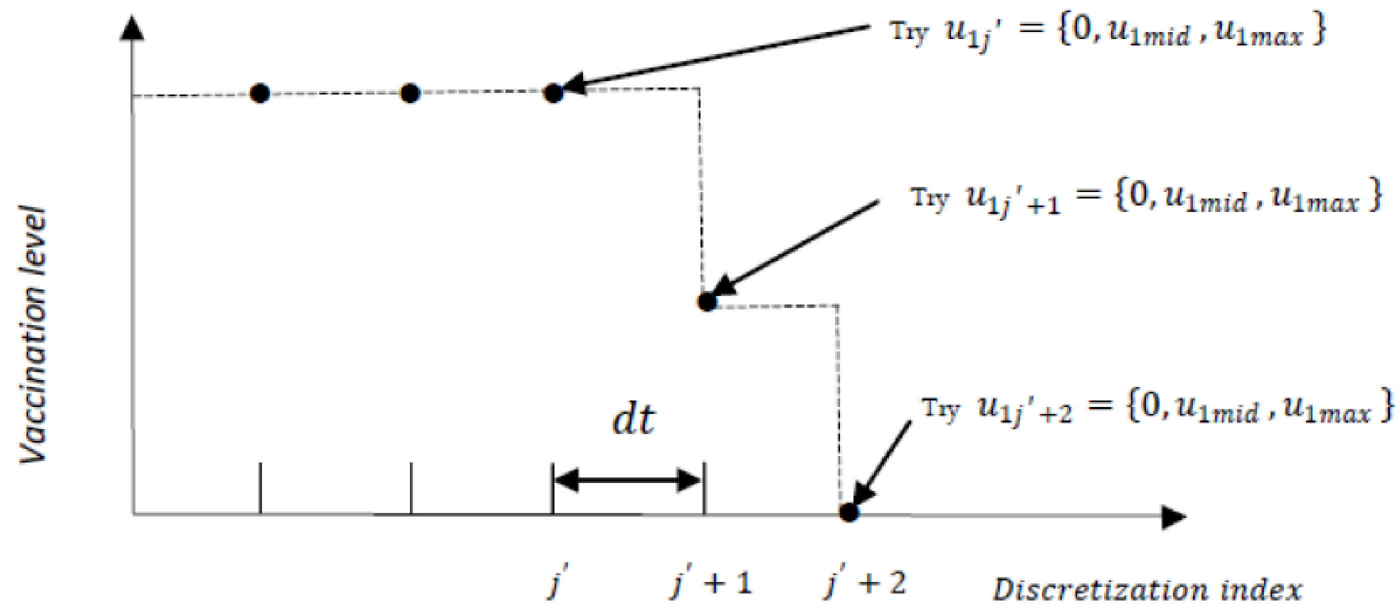


Figure 6: Vaccination strategy modification in stage III of algorithm.

Results: Cost decreases from stages I, II, III

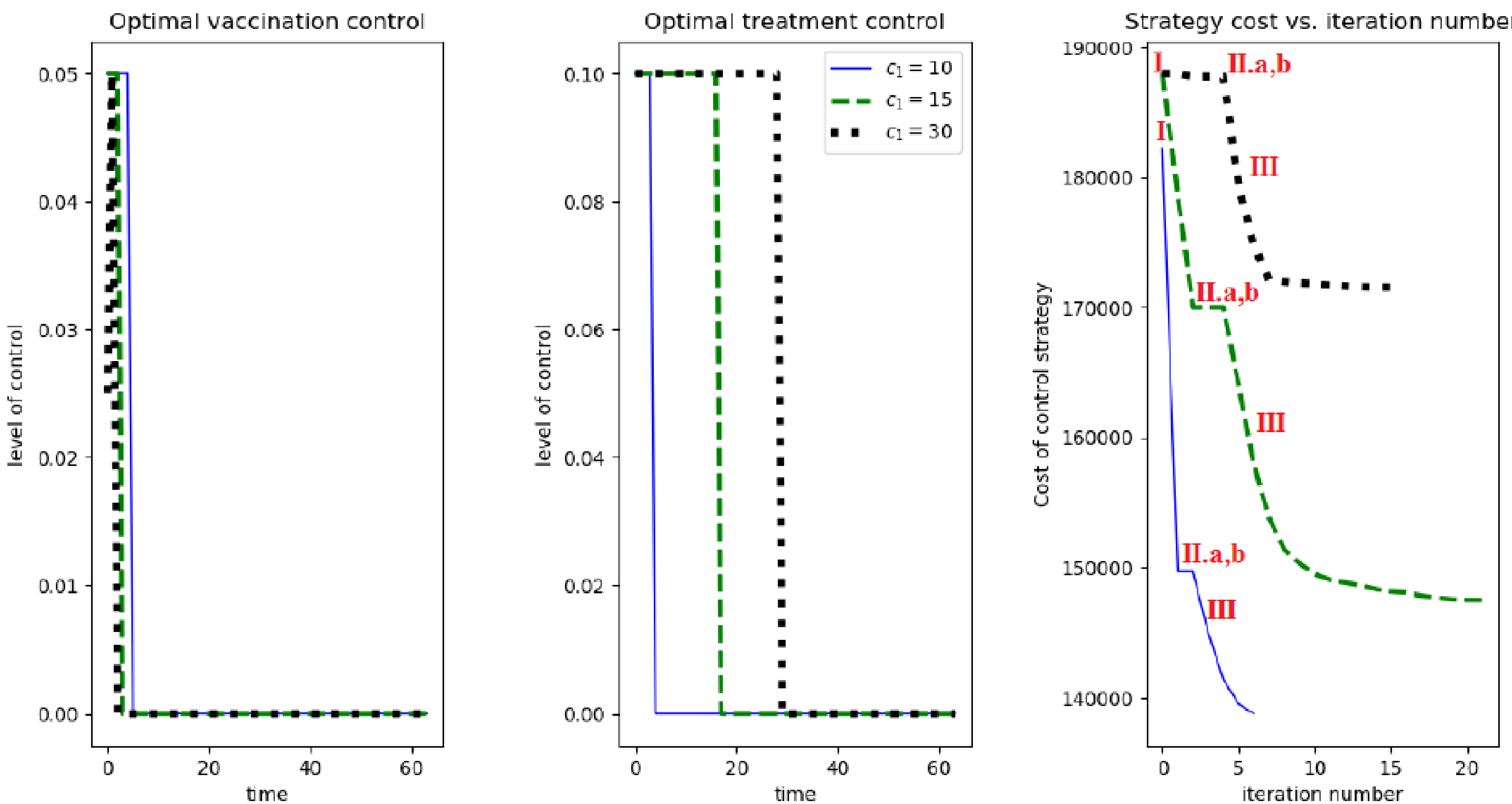


Figure 9. Optimal vaccination, treatment controls and strategy cost for different values of c_1.

Summary: Lessons Learned

- It is important to try to make your results interesting to non-specialists.
- If you want others to believe your results, use state-of-the-art models developed by others instead of creating your own model.
- Use numerical rather than analytical methods, because classical methods make too many assumptions that do not fit the real case.
- Learn to write efficient code in python
- To have an impact you must explain your results to non-specialists. Put a lot of thought into data representation and visualization.

Resources

The SIR Model for Spread of Disease – Introduction (*Mathematical Association of America)*

https://www.maa.org/press/periodicals/loci/joma/the-sir-model-for-spread-of-disease-introduction

An introduction to compartmental modeling for the budding infectious disease modeler

Letters in Biomathematics 5(1):195-221

https://www.researchgate.net/publication/327067041_An_introduction_to_compartmental_modeling_for_the_budding_infectious_disease_modeler

Thank you for your attention

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شكرا على الاهتمام

Merci pour votre attention

感谢您的关注

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Asante kwa mawazo yako

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Ngiyabonga ukulalela kwenu

Enkosi ngosinaka ixehsa lakho

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Predictive Modelling of COVID-19 Pandemic Evolution in Nigeria

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Abstract

COVID-19 is a pandemic that has defined human life, shutting down economic activities. Various measures have been implemented to curb its spread. However, in many places, confirmed cases have continued to increase. Many believe that unless its vaccine is discovered, the pandemic has come to stay. This article aims to develop a model for the evolution of the total confirmed cases, which in the early stage increase exponentially and in the later stage flatten out.

Following the balance equation modelling and employing assumptions similar to the typical pandemic modelling, an exponential model was developed. In order to describe the whole trajectory of the pandemic, following the general trend of COVID-19 data, the exponential model was modified with an inverse exponential expression, comparable to the Arrhenius Equation in chemical reaction engineering. After parameter estimation, the resulting model was validated using data from Italy and Nigeria. The model predictions compare reasonably well with the data.

The model was then employed to predict the future of COVID-19 in Nigeria. The final equilibrium total confirmed cases would be 81,292 and the time for the country to record very low new daily cases would be in March 2021.

1. Introduction

Coronavirus disease 2019, popular known by its abbreviated form COVID-19 is a recent virus infection and a global pandemic currently ravaging the whole world (Ong et al., 2020; WHO, 2020[a]). As of 16 August, 2020, the global confirmed cases of people who had contracted the disease were in excess of 21 million; more than 13.5 million had recovered and nearly 774,000 dead, leaving slightly above 7 million people as active carriers of the virus and COVID-19 patients (WHO, 2020[b]). These cases were reported in 188 countries out of the 195 countries of the world in less than a year, rising from the few initial cases of the disease reported in Hubei City of China in November 2019 (CSSE, 2020; Aljazeera, 2020). Research shows that the initial cases in China occurred by zoonosis, bat-to-human transmission, while first cases in other parts of the world occurred by cross border migration, classified as imported cases (Sun et al, 2020). Then the imported cases transmitted locally before metamorphosing into community transmission. After about 300 days since inception, new covid-19 cases in China occurred sporadically and in clusters and the country was approaching the end of the pandemic (WHO, 2020c). Countries such as New Zealand have almost attained the status of being covid-19 free while some countries in Africa and Europe experience prolonged phase of community transmission (Rohatgi, 2020; Cobore et al., 2020). Qualitatively, the evolution of the pandemic is described as no cases, zoonosis or imported cases, local and community transmissions, cluster of cases, sporadic cases and COVID-19 free status.

Understanding the disease evolution is one of the critical strategies for the prevention and control of the pandemic, the reduction of related mortality and minimization of economic impact; prevention and control measures to be adopted depend on the evolution stage (WHO, 2020d and e). These measures include active case finding, signs and symptoms checking, laboratory testing, contact tracing, and patients' isolation. Others are travel restrictions, border closures, clinical management, and community engagement. At the inception of the pandemic, China deployed active case finding, signs and symptoms checking, and laboratory testing to locate COVID-19 active cases in Hubei City where the disease was contracted via zoonotic transmission (WHO, 2020d). Several countries enforced self-isolation, isolation centres, and contacts' tracing for the imported cases, their relatives and associates. Border closures also controlled the imported cases while territory isolation and total lockdown restricted local transmission. The community engagement, a further measure to curb local and community transmission, involves aggressive campaigns and publicity for social distancing, use of sanitizers, face masks and infrared thermometers. With the introduction of these various measures, analyses of epidemiologic data and transmission dynamics would show effectiveness in curbing and even stamping out the pandemic.

Mathematical modelling is a tool that can be used to describe pandemic transmission dynamics; analyse impact and effectiveness of various prevention and control measures; and estimate key transmission and severity parameters (Wang et al., 2020). Since the beginning of the pandemic, various studies on mathematical modelling have been published. Some focus on statistical and stochastic analyses of epidemiologic data (Sun et al., 2020) while some draw analogies for descriptive analyses of the evolutionary trend as obtained for example in the SEIR (Susceptible-Exposed-Infectious-Recovered) modelling (Sun and Li, 2020; Nwalili et al., 2020). The latter modelling is a variant of balance equation modelling widely employed in chemical reactions engineering, where rate constant values determine rates of reaction and increase with temperature according to the Arrhenius equation (Fogler, 2004). As COVID-19 data and trend change rapidly, new studies based on recent information and modelling, not only valid for the early stage of the pandemic but also for the entire time trajectory, should be sustained.

Key COVID-19 epidemiologic data include total confirmed cases, recovered cases, daily recorded cases and death cases. Others include tested samples, days since the last recorded case and transmission classification (WHO, 2020a). The most important variable is the total confirmed cases; it shows the proportion of the population infected with the disease as well as a measure of the pandemic impact on the economy. Further, the total confirmed cases comprise active, dead and recovered cases; alongside epidemiologic statistical data, these component cases can be estimated from total confirmed cases. Also, with the total confirmed cases represented by a function of time, its first derivative becomes an equation that would yield new recorded cases. A model for the total confirmed cases would therefore be informative in describing key indicators of the pandemic.

The objective of this article is to develop a model for the COVID-19 pandemic using the balance equation modelling in chemical reactions engineering and considering two components of the population of a country: total confirmed cases and the rest. From the world's COVID-19 data as a whole and those of individual countries and territories as of August 16, 2020, proportions of total confirmed cases were less than 1 % of total human populations (WHO, 2020a; UN, 2020). In mathematical modelling therefore, the uninfected population can be assumed to be constant. Following this assumption, the developed model is subsequently simplified, yielding an exponential equation. As its name suggests, with a constant-value coefficient, this equation diverges exponentially, capturing only the early part of the pandemic, but not the whole pandemic time trajectory. Instead of a constant value, a variable coefficient that changes with time, to capture various infection prevention and control interventions at different times aimed at flattening the pandemic time curve, is introduced. The time function can assume different forms. However, its introduction should yield a model that can describe the entire pandemic time trajectory as witnessed in countries nearing the end of COVID-19. Using data from these countries, an inverse exponential time function able to arrest the exponential divergence and vanish it at infinite time, was justified, validated and used in the model. This model was subsequently employed for the Nigerian COVID-19 pandemic to determine key indicators such as the expected final equilibrium value of the total confirmed case, the maximum daily recorded case and the corresponding time; the estimated time for the daily recorded cases to be insignificant (for example, considering values in the order of one new case in 10 million people).

Thus, the article comprises six sections. Section 2 follows this introductory section and develops an exponential model capable of describing the early part of the pandemic. Section 3 modifies the exponential model and estimates parameter values in describing the pandemic using COVID-19 data from China and Italy. Subsequently, Section 4 employs COVID-19 data from Nigeria to validate the modified model. Looking into the future, Section 5 predicts the end of COVID-19 in Nigeria. Finally, Section 6 reports the conclusions.

2. Exponential model

In this section, we adopt the balance modelling approach to develop a model for the total confirmed cases of COVID-19. These cases comprise active, recovered and dead cases. By implication, we divide the human population into two compartments: healthy population (those who have never contracted the disease) and confirmed cases. These two compartments are those simply required to develop an expression for total confirmed cases. The compartments can be three in the SIR (Susceptible-Infectious-Recovered) modelling or four in the SEIR modelling (Beckley et al., 2013).

As in the latter, the following assumptions apply. The population in each compartment is described by a continuous function. Furthermore, the region under investigation is closed. Thus, cross-border migration is neglected. Finally, each population evolves by interacting with the other.

Thus, the number of confirmed cases is described as:

$$\frac{dP}{dt} = k_F P P_H \tag{1}$$

where P is the number of confirmed cases per unit area; P_H is the area density of the healthy population; and k_F (measured in m^2 per day) is the pandemic spreading rate constant.

On the other hand, the healthy population, which decreases by interacting with confirmed cases of COVID-19 and increases by the cumulative effect of other factors such as birth and death, is described as:

$$\frac{dP_H}{dt} = -k_F P P_H + k_G P_H \tag{2}$$

where k_G is the demographic population growth rate. While the coronavirus disease decreases the healthy population by the first term on the right-hand side of Eq. (2), by the second term, population growth increases it. Although it has been declining, the world population growth rate for the period 2010 to 2015 was still positive at 1.2% (UN, 2016). The healthy population growth due to other factors and the decrease due to the impact of COVID-19 are therefore assumed to cancel out, thereby making the healthy population constant. Thus, Eq. (2) becomes:

$$\frac{dP_H}{dt} = -k_F P P_H = 0; \qquad P_H = B\ (a\ constant) \tag{3}$$

By substituting Equation (3) into equation (1), the latter can be integrated to yield:

$$\ln(P/P_0) = k_F B t; P_F = P_0 e^{(Kt)} \tag{4}$$

where $K = k_F B$; it is a pseudo-exponential rate constant. Its value indicates how fast the disease spreads.

Eq. (4) is a simple population growth model, illustrating that the confirmed cases of COVID-19 increase exponentially for any positive value of K and suggesting that eventually, everyone in the healthy population becomes infected. However, due to various infection prevention and control measures, the disease exponential spread can be reduced such that more than 90 % of the population escape the infection. In order to reflect this in the equation, K must be modified so that its value vanishes at infinite time. The modification can be written as:

$$\frac{dP}{dt} = f(t)\,P\,; \qquad f(t) = \frac{dP/dt}{P} \tag{5}$$

where $f(t)$ is a function of time with k_F and B embedded.

A further analysis of COVID-19 data will help in determining the functional form of $f(t)$. This analysis is performed in the following section.

3. Modified model

In this section, we modify the exponential model developed in Section 2 to describe the entire trajectory of the COVID-19 pandemic and then employ it for the predictions of COVID-19 cases in Italy. In the following, we first demonstrate the equation that best describes the evolution of the pandemic.

Figure 1 shows the time evolution of COVID-19 confirmed in China (WHO, 2020[a]). This profile is typical of the disease. In the early part, dP/dt at the numerator on the y-axis variable ($dP/(Pdt)$) is finite and begins to rise while P at the denominator is small. Their combination yields a large value. As P increases exponentially, the y-axis variable decreases exponentially with time, thus the profile.

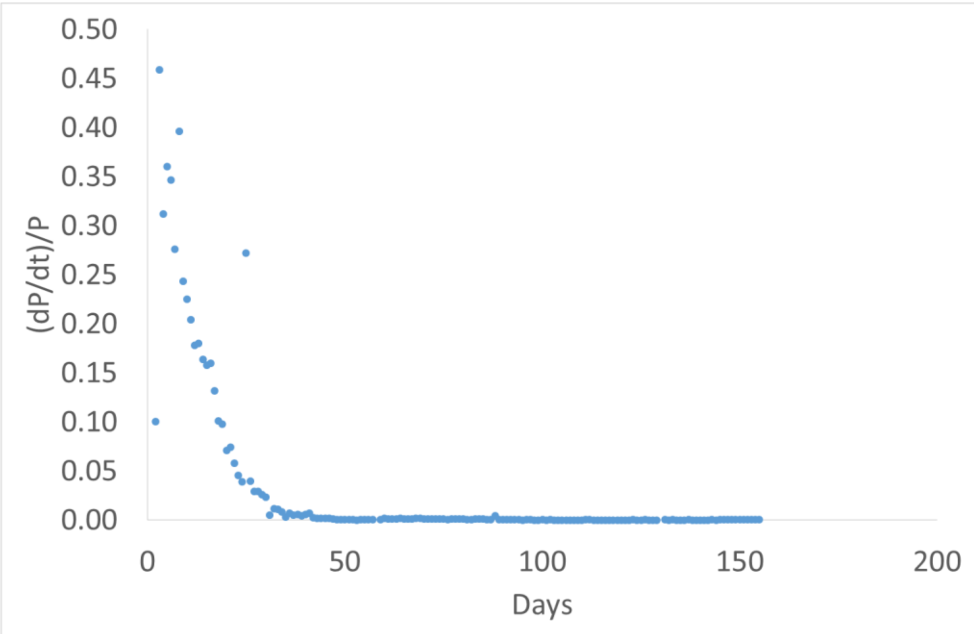


Figure 1: Rate of Covid-19 cases with time using data from China. Data from WHO (2020[a]).

Thus, for the modelling of the spread of COVID-19, we write:

$$\frac{dP}{dt} = K_0 e^{-at} P \quad (6)$$

where K_0 (measured in per day) is the disease infection rate constant, and a (measured in per day) is an exponential constant, which accounts for the changes in the rate constant with time. The rate constant along with the exponential term resembles the Arrhenius Equation in chemical kinetics for the effect of temperature on reaction rates; refer to Fogler (2004) or other texts on chemical reactions engineering for details.

The modified model in Eq. (6) is the rate of change of the confirmed cases with time. By integrating this equation, we can obtain the total confirmed cases of COVID-19 at any time. As it is, the equation yields the new recorded cases at any time. Further differentiation yields an expression for the maximum new recorded case. The integration is obtained as follows.

Rearranging Eq. (6), we have:

$$\frac{dP}{P} = K_0 e^{-at} dt \quad (7)$$

Integration yields:

$$\ln P = -\frac{K_0}{a} e^{-at} + C \quad (8)$$

where C is the integration constant, obtained as follows:

$$C = \ln P_0 + \frac{K_0}{a} e^{-at_0} \quad (9)$$

where t_0 can be taken as the first day of the index case P_0, the initial condition. Thus, the integral is:

$$P = e^{\left[\frac{K_0}{a}(e^{-at_0} - e^{-at}) + \ln P_0\right]} \quad (10)$$

On the other hand, the differentiation of Eq. (6) yields:

$$\frac{d}{dt}\left(\frac{dP}{dt}\right) = \frac{d}{dt}(K_0 e^{-at} P); \frac{d^2P}{dt^2} = -K_0 a e^{-at} P + K_0 e^{-at} \frac{dP}{dt}; \frac{d^2P}{dt^2} = -K_0 a e^{-at} P + {K_0}^2 e^{-2at} P$$

(11)

At the peak increase in daily confirmed cases corresponding to time t_P, Eq. (11) is zero as:

$$\frac{d^2P}{dt^2} = -K_0 a e^{-at_P} P + {K_0}^2 e^{-2at_P} P = 0; a = K_0 e^{-at_P} \quad (12)$$

The time for the peak daily increase is:

$$t_P = \frac{1}{a} \ln \frac{K_0}{a} \quad (13)$$

Values of K_0 and a are required to use this model. These can be obtained by fitting the model to some data and validating it with some other data as follows. Rearranging Eq. (6), we obtain:

$$\ln\left(\frac{dP/dt}{P}\right) = \ln K_0 - at \quad (14)$$

COVID-19 data with P as total confirmed cases with time are required to calculate the values of K_0 and a. For this reason, we employ COVID-19 data for the first 150 days of the pandemic in Italy, a country that has gone through almost all the stages in the evolution of the pandemic (WHO, 2020[a]),.

Data for the first 100 days are used for parameter estimation while the remaining data are used for model validation. For the parameter estimation, Figure 2 shows a linear correlation of the data.

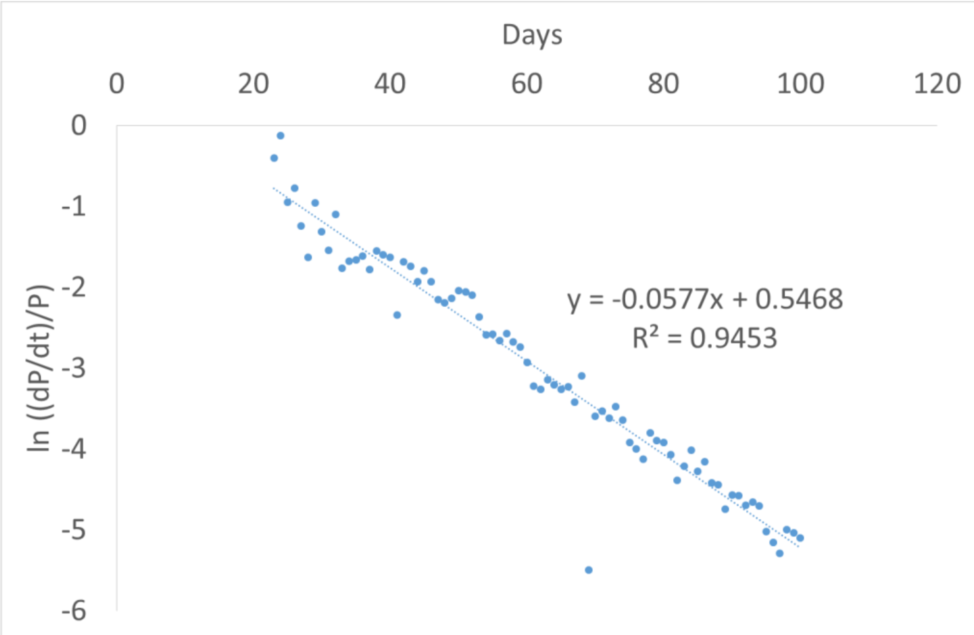


Figure 2: Parameter estimation for modified model 1 using the data from Italy. [Source: WHO 2020[a]]

In comparison with Eq. (14),

$$a = 0.0577; K_0 = e^{0.5468} = 1.728 \tag{15}$$

Using these values, the evolutions of P and dP/dt for the remaining data can be estimated. While P is given by Eq. (10), dP/dt in terms of time can be obtained by substituting the latter equation for P in Eq. (6) as follows.

$$\frac{dP}{dt} = K_0 e^{-at} e^{\left[\frac{K_0}{a}\left(e^{-at_0} - e^{-at}\right) + \ln P_0\right]} \tag{16}$$

For the COVID-19 data of Italy, when $t_0 = 1$, $P_0 = 2$. Thus:

$$P = 2e^{\frac{K_0}{a}\left(e^{-a} - e^{-at}\right)} = 2e^{9.4766\left(e^{-0.0577} - e^{-0.0577t}\right)} \tag{17}$$

and

$$\frac{dP}{dt} = 2K_0 e^{-at} e^{\frac{K_0}{a}\left(e^{-a} - e^{-at}\right)} = 1.0936 e^{-0.0577t} e^{9.4766\left(e^{-0.0577} - e^{-0.0577t}\right)} \tag{18}$$

For the validation of the Eqs (17) and (18), we employed the remaining COVID-19 data of Italy. Figures 3A and B show how the predicted total confirmed cases and daily confirmed cases from modified model 2 compare with the data.

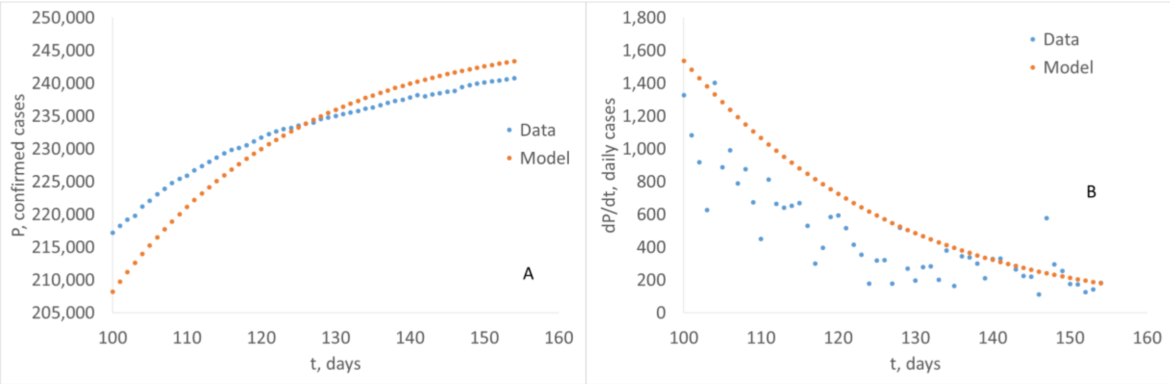


Figure 3: Validation of the modified model 2 using 100 to 150 days of Italy Covid-19 data and the correlation parameters obtained from the first 100 days – A: total confirmed cases and B: daily confirmed cases. Data from WHO (2020[a]).

By manually fitting the model to the data, better predictions were obtained with $a = 0.04257$ and $K_0 = 0.5209$. See Figure 4. These predictions suggest that the original values were at a suboptimal point.

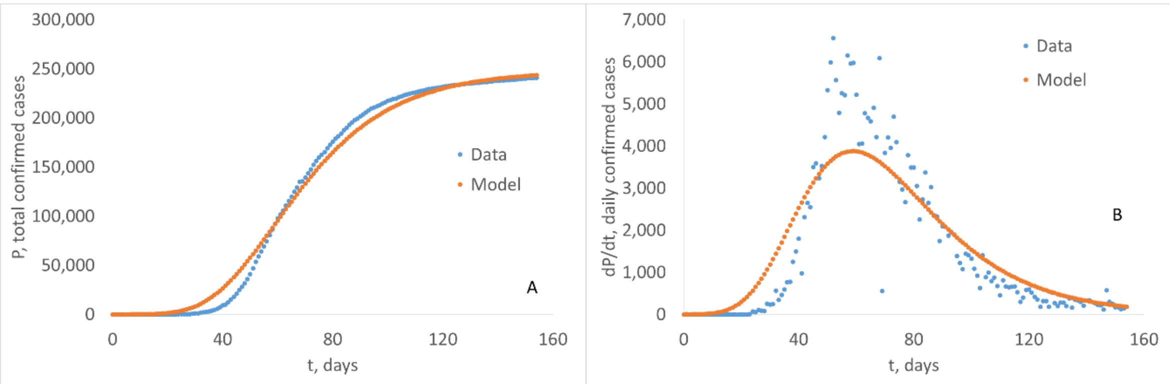


Figure 4: Validation of the modified model 2 using Italy Covid-19 data and the parameters manually adjusted to $a = 0.04257$ and $K_0 = 0.5209$ – A: total confirmed cases and B: daily confirmed cases. Data from WHO (2020[a]).

4. Model validation

Above procedures were followed to predict Nigeria's COVID-19 cases. The first 100 data points were used to obtain the parameters: K_0 and a in Eq. (14). Figure 5 shows the correlation.

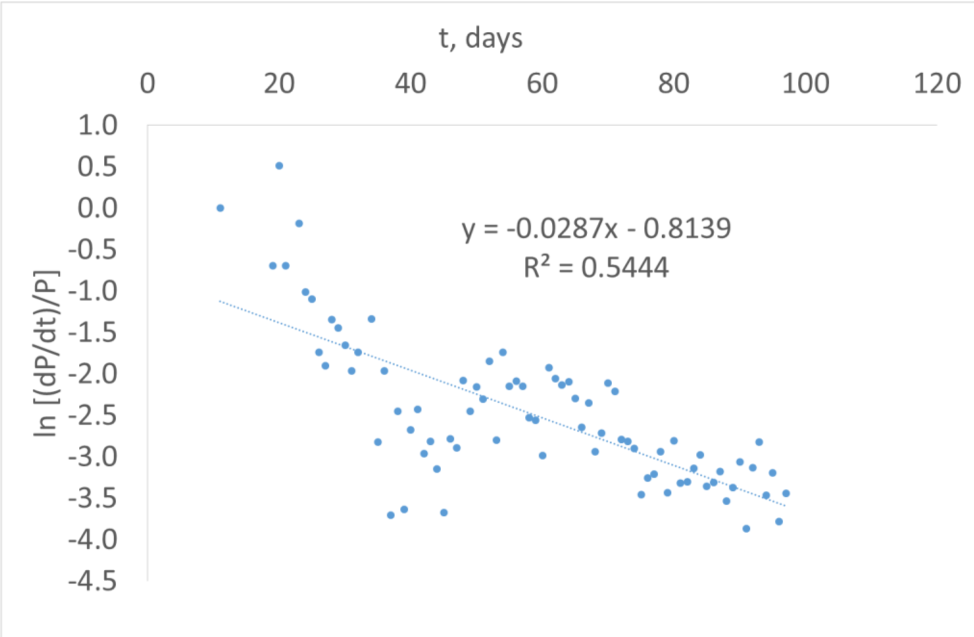


Figure 5: Parameter estimation for modified model 1 using the data from Nigeria. Data from NCDC (2020).

The linear equation, however, poorly correlates the data.

In comparison with Eq. (14),

$$a = 0.0287; K_0 = e^{-0.8139} = 0.4431 \quad (19)$$

For the COVID-19 data of Nigeria, when $t_0 = 1$, $P_0 = 1$. Thus:

$$P = e^{\frac{K_0}{a}(e^{-a}-e^{-at})} = e^{15.439(e^{-0.0287}-e^{-0.0287t})} \quad (20)$$

and $\frac{dP}{dt} = K_0 e^{-at} e^{\frac{K_0}{a}(e^{-a}-e^{-at})} = 0.4431e^{-0.0287t} e^{15.439(e^{-0.0287}-e^{-0.0287t})}$ (21)

For the validation of the Eqs (20) and (21), we employ COVID-19 data for 100 to 125 days since the index case in Nigeria. Figures 5A and B show how the predicted total confirmed cases and daily confirmed cases from the modified model compare with the data.

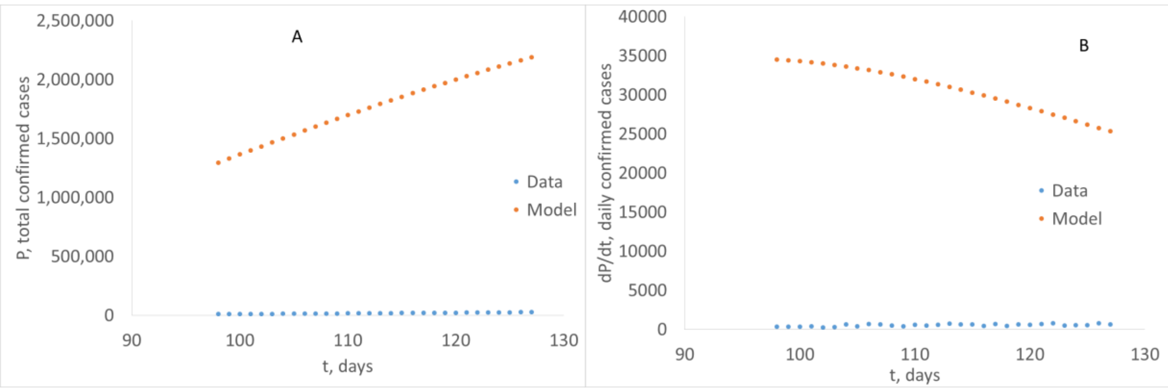


Figure 6: Validation of the modified model 2 using 80 to 130 days of Nigeria Covid-19 data and the correlation parameters obtained from the first 80 days – A: total confirmed cases and B: daily confirmed cases. Data from NCDC (2020).

Because of the poor correlation in Figure 5, the predictions in Figures 6A and B are poor. We therefore resort to manual fitting. The initial values for the manual fitting were

$$a = 0.01829 \text{ and } K_0 = 0.1867 \quad (22)$$

These initial values were obtained by assuming the peak of the curve of dP/dt being the highest recorded new case of 790 occurring at $t_P = 127\ days$ with $P = 27110$ and $P_0 = 1$ from the Nigeria COVID-19 data and solving the nonlinear equations of Eqs. (10) and (13) simultaneously.

With the values in Eq. (22), Figures 7A and B show how the predicted total confirmed cases and daily confirmed cases from the modified model compare with the data.

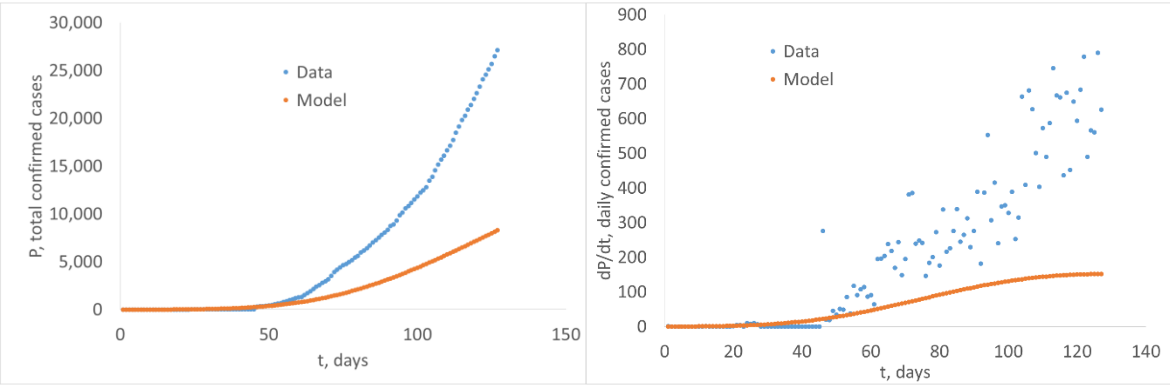


Figure 7: Validation of the modified model using Nigeria Covid-19 data and the parameters obtained by simultaneously solving t_P and $P\ (t = 127; P = 27110)$ – A: total confirmed cases and B: daily confirmed cases. Data from NCDC (2020).

By manually fitting the model to the data, better predictions were obtained with $a = 0.01829$ and $K_0 = 0.2106$. See Figure 8.

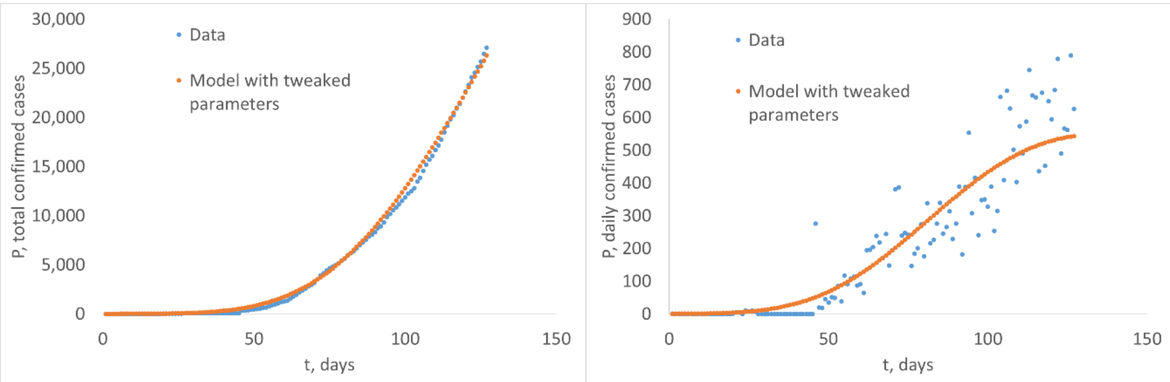


Figure 8: Validation of the modified model 2 using Italy Covid-19 data and the parameters tweaked to $a = 0.01829$ and $K_0 = 0.2106$ – A: total confirmed cases and B: daily confirmed cases. Data from NCDC (2020).

As shown, we have been able to model the covid-19 cases for Nigeria. This model with the parameter values can be used to predict the evolution daily cases and total confirmed cases.

5. Future predictions

This section further tests model predictions of the Nigerian COVID-19 data using the month of July and analyses the model for key indicators of the pandemic.

In predicting the Nigerian COVID-19 data beyond 125 days after the index case, the model in Eq. (20) along with the parameter values in Eq. (22) were first used for July 03 – August 02, 2020. Figure 9 shows the comparison between the data and model predictions.

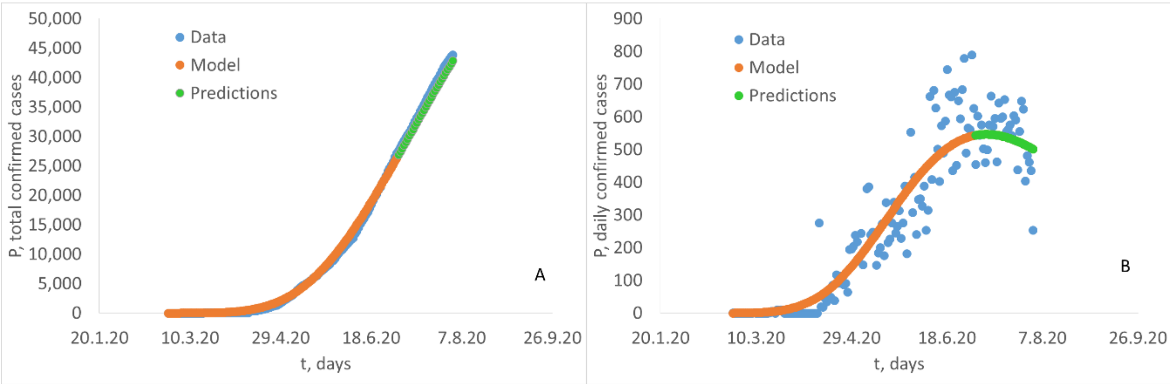


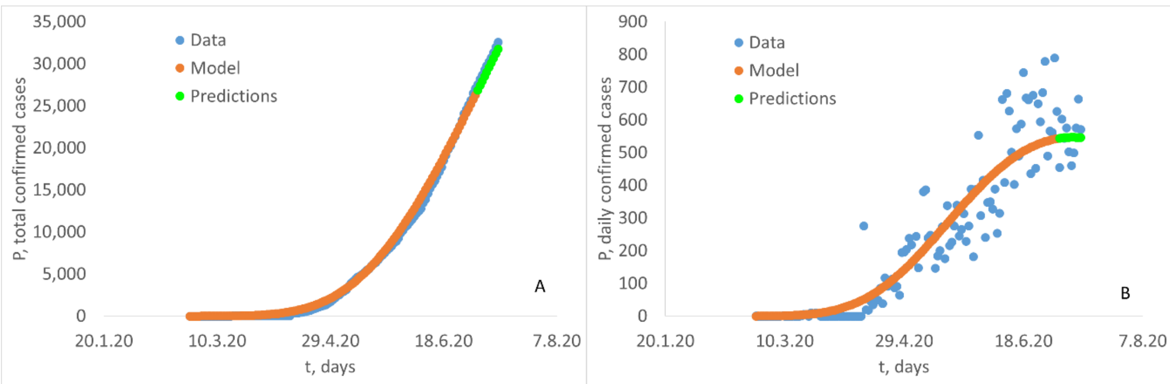
Figure 9: Model predictions of the COVID-19 cases in Nigeria for the period July 03 – August 02, 2020 highlighted in green; A: Total confirmed cases, B: New daily recorded cases. Data from NCDC (2020).

The model predictions are almost identical with the data for the total confirmed cases and have good fitting with the data for the daily recorded cases. To fit the new daily recorded cases better, a sinusoidal function should be included in the model as the data appear to oscillate around the predictions. However, this inclusion would lead to a more complex model.

In order to further track the predictions, the data are analysed weekly as follows:

Week 1 (July 03 – July 12)

Eqs (20) – (22) were subsequently used to predict the total confirmed cases and new daily recorded cases in Nigeria for the period 121 – 130 days after the index case, corresponding to July 03 – July 12, 2020. Figure 10 shows the comparison between the data and model predictions.



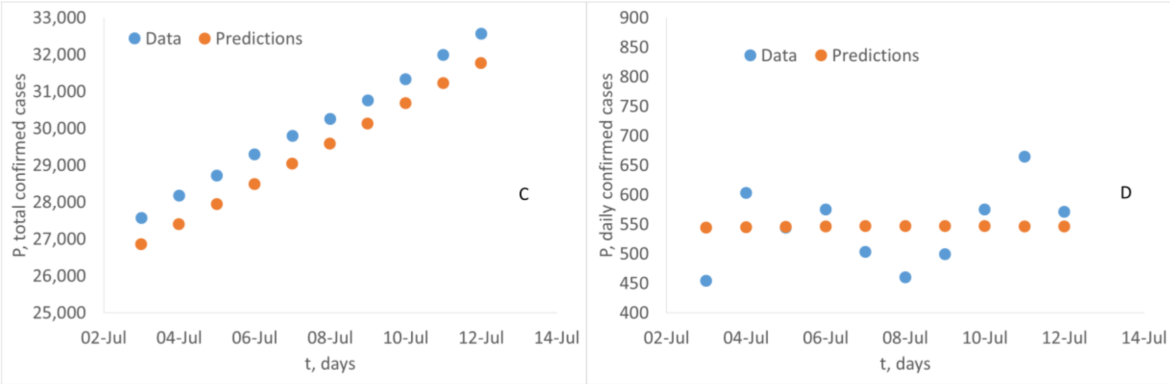


Figure 10: Model predictions of the COVID-19 cases in Nigeria for the period July 03 – July 12, 2020 highlighted in green; A: Total confirmed cases, B: New daily recorded cases; C: Predictions in A magnified, D: Predictions in B magnified. Data from NCDC (2020).

As shown, for the total confirmed cases, the predictions increase in a similar pattern as the real data. On the other hand, for the new daily recorded cases, the model predictions do not change significantly within the period of observation. The predictions appear to be almost at a constant value while real data oscillate around it (see quadrants C and D in Figure 10).

While keeping the value of K_0 constant at 0.2106, to obtain better predictions of the total confirmed cases with values almost identical with the real cases, the value of the parameter a was manually adjusted to 99.65% of the original value, i.e. $a = 0.01823$. The value of K_0 is kept constant as the spreading rate function by analogy compares with the Arrhenius Equation, where the pre-exponential factor must be a constant value. Figure 11 shows the new predictions.

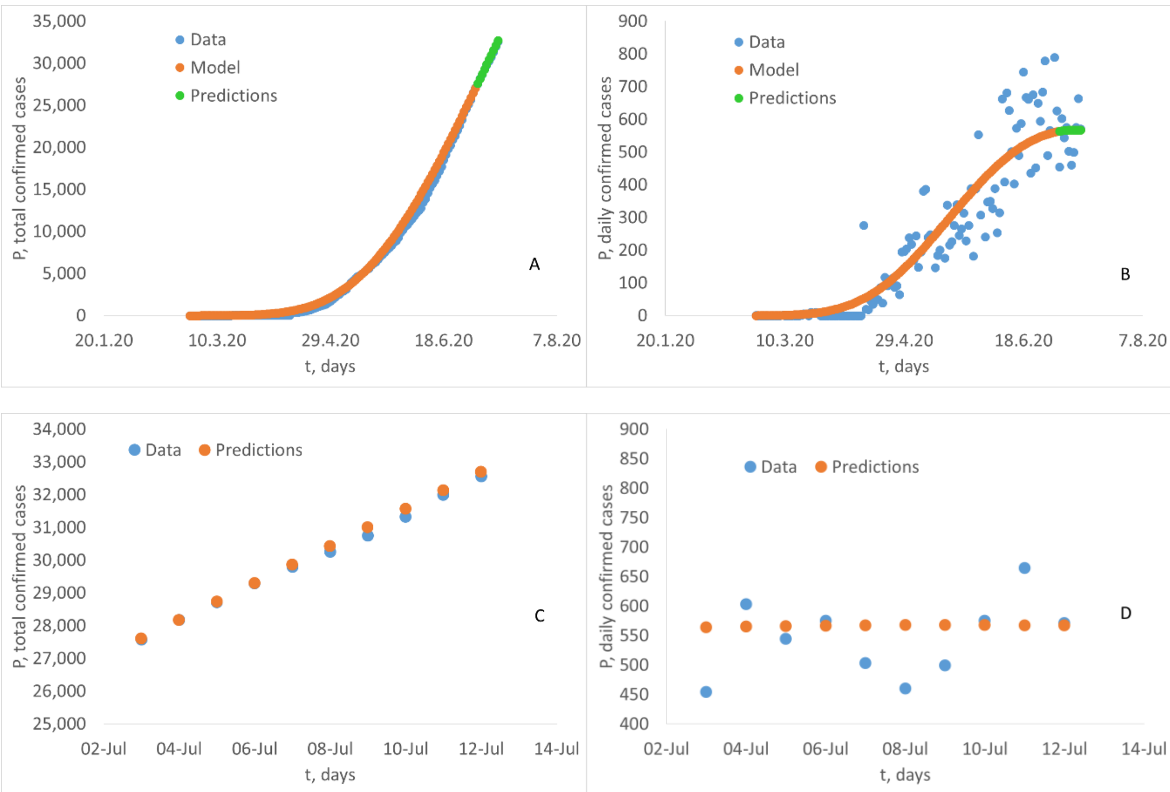


Figure 11: New model predictions of the COVID-19 cases in Nigeria for the period July 03 – July 12, 2020 highlighted in green with $a = 0.01823$; A: Total confirmed cases, B: New daily recorded cases; C: Predictions in A magnified, D: Predictions in B magnified. Data from NCDC (2020).

Week 2 (July 13 – July 19)

While the value of K_0 was kept constant at 0.2106 corresponding to its fitting value of the first 125 days, and that of a being 0.01823, corresponding to its fitting value for Week 1 above, the model was used to predict the Nigerian COVID-19 cases for week 2 in the period July 13 – July 19, 2020. Figure 12 shows the comparison between the data and model predictions.

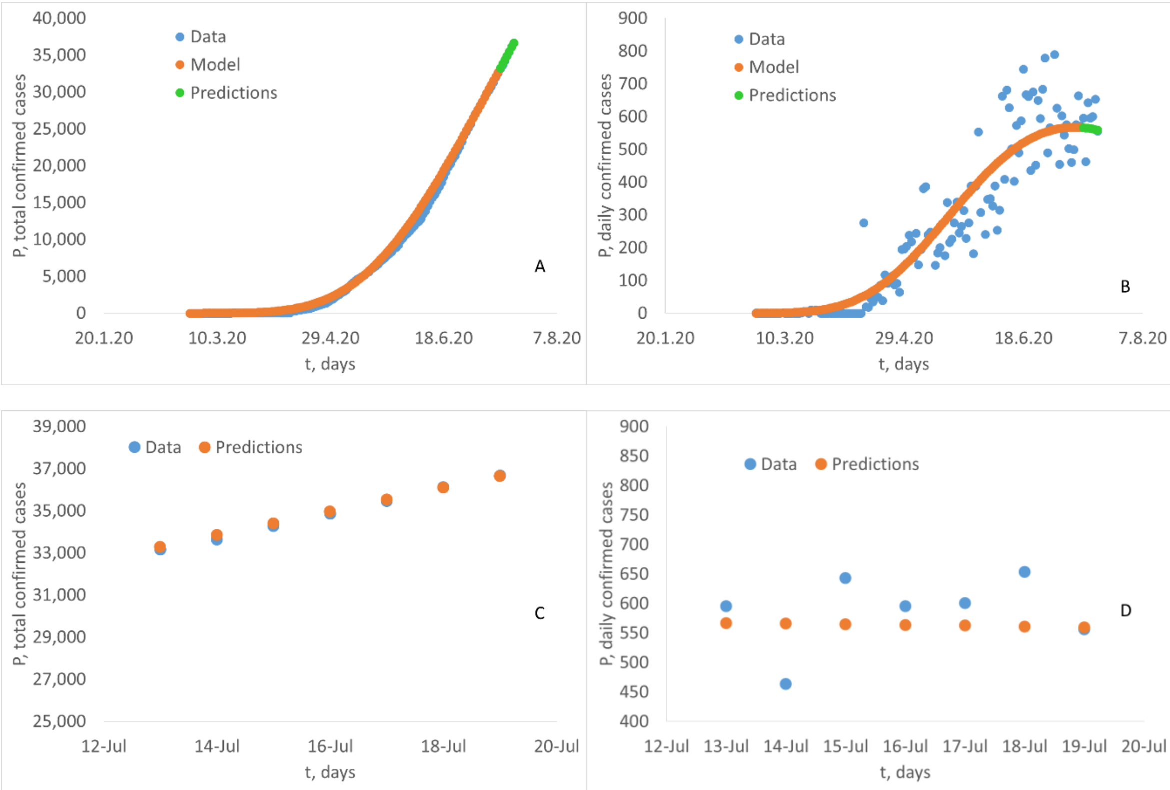


Figure 12: New model predictions of the COVID-19 cases in Nigeria for the period July 13 – July 19, 2020 highlighted in green with $a = 0.01823$; A: Total confirmed cases, B: New daily recorded cases; C: Predictions in A magnified, D: Predictions in B magnified. Data from NCDC (2020).

As shown, for the total confirmed cases, the predictions show excellent agreement with the data. On the other hand, for the daily recorded cases, the model predictions do not change significantly within the period of observation while the data continue to display an oscillatory pattern.

Week 3 (July 20 – July 26) and Week 4 (July 27 – August 02)

For Weeks 3 and 4, the procedure and parameter values for week 2 above apply. Figures 13 and 14 show the comparison between the data and model predictions.

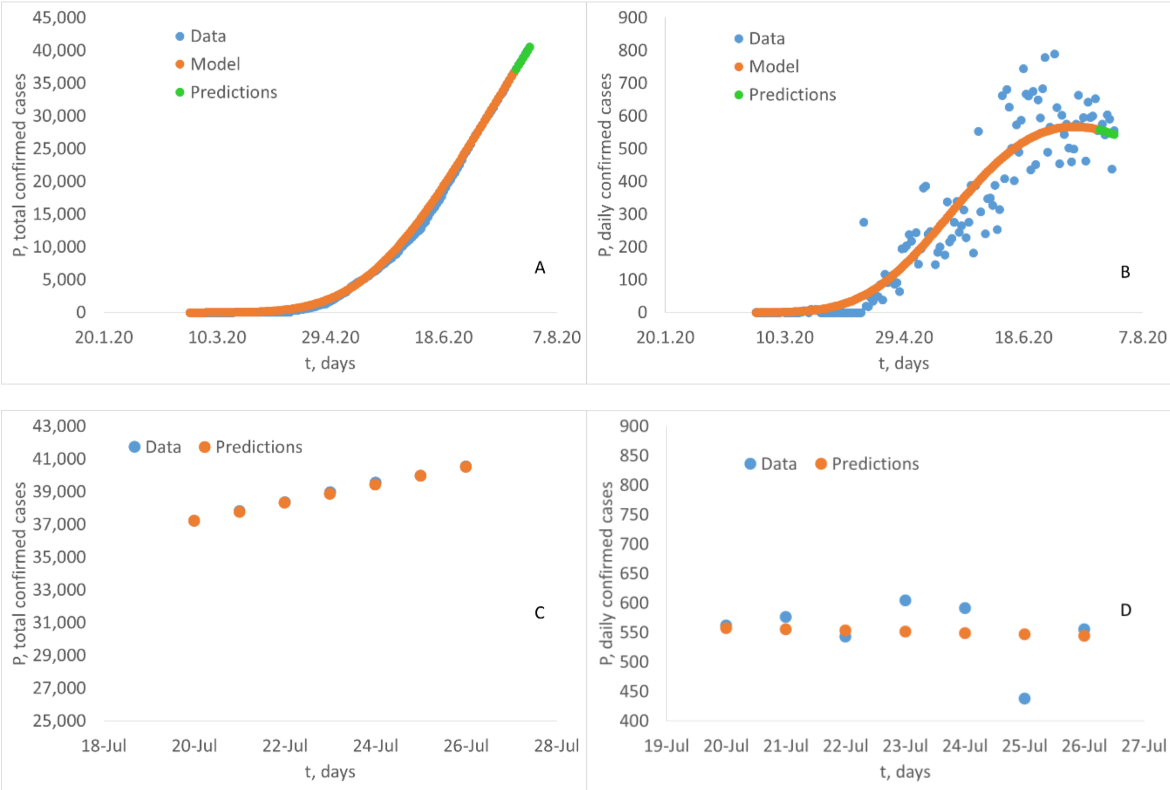


Figure 13: New model predictions of the COVID-19 cases in Nigeria for the period July 20 – July 26, 2020 highlighted in green with $a = 0.01823$; A: Total confirmed cases, B: New daily recorded cases; C: Predictions in A magnified, D: Predictions in B magnified. Data from NCDC (2020).

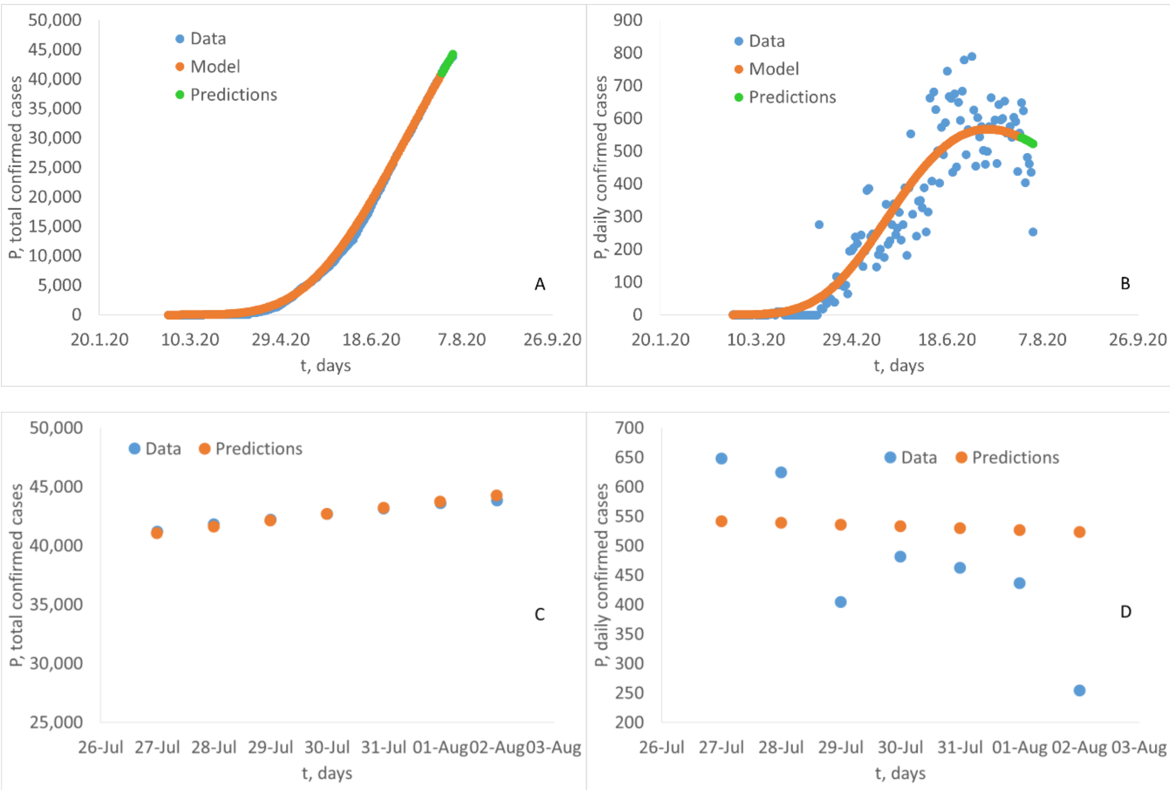


Figure 14: New model predictions of the COVID-19 cases in Nigeria for the period July 27 – August 02, 2020 highlighted in green with $a = 0.01823$; A: Total confirmed cases, B: New daily recorded cases; C: Predictions in A magnified, D: Predictions in B magnified. Data from NCDC (2020).

The model predictions are identical with the data for the total confirmed cases and have excellent fitting with the data for the new daily recorded cases. For this week, the decline in the predicted daily recorded cases is now obvious unlike the apparent horizontal profile of the previous weeks.

Key indicators of the Pandemic

The model can be used to determine the following: the final equilibrium value of P as $t \to \infty$; the maximum new daily recorded case and the corresponding time; the estimated time for the daily recorded cases to be insignificant (that is, 1 new case in 10 million people).

For the final equilibrium value, using the parameters in Eq. (22), we substitute $t = \infty$ into Eq. (20) as:

$$P = P_0 e^{\frac{K_0}{a}(e^{-a} - e^{-at})} = e^{\frac{0.2106}{0.01829}(e^{-0.01829} - e^{-0.01829\times\infty})} = 81{,}292 \ confirmed\ cases$$

For the maximum daily recorded case, we differentiate Eq. (21) and then equate to zero (the necessary condition for an optimum point):

$$\frac{d}{dt}\left(\frac{dP}{dt}\right) = \frac{d}{dt}(K_0 e^{-at} P); \frac{d^2P}{dt^2} = -K_0 a e^{-at} P + {K_0}^2 e^{-2at} P \quad (23)$$

$$\frac{d^2P}{dt^2} = -K_0 a e^{-at} P + {K_0}^2 e^{-2at} P = 0; a = K_0 e^{-at} \quad (24)$$

$$t = \frac{\ln\frac{a}{K_0}}{-a} = \frac{\ln\frac{K_0}{a}}{a} \quad (25)$$

Substituting the parameter values,

$$t = \frac{\ln\frac{0.2106}{0.01829}}{0.01829} = 133.6 \cong 134\ days$$

The corresponding maximum daily rise is:

$$\frac{dP}{dt} = K_0 P_0 e^{-at} e^{\frac{K_0}{a}(e^{-a} - e^{-at})} = 0.2106 \times 1 \times e^{-0.01829\times133.6} e^{\frac{0.2106}{0.01829}(e^{-0.01829} - e^{-0.01829\times133.6})}$$

$$= 546.9 \cong 547\ cases$$

Finally, for the estimated time for the daily recorded cases to be very low (that is 1 new per 10 million people or equivalently, below twenty new cases per day in Nigeria), the predicted P is computed until its value is below 20. The corresponding time is 369 days after the index case, which falls on March 01, 2021. Figures show the full pandemic evolution of COVID-19 in Nigeria.

The model reported in the previous reports was used for the predictions of the COVID-19 cases in Nigeria. Figure 15 shows the full pandemic evolution. 'A' shows the total confirmed cases while 'B' shows the new daily cases. In the former, the model predictions are identical to the data while in the latter, the model predictions describe the new cases quite well, thus informing as reasonable the predicted full pandemic evolution.

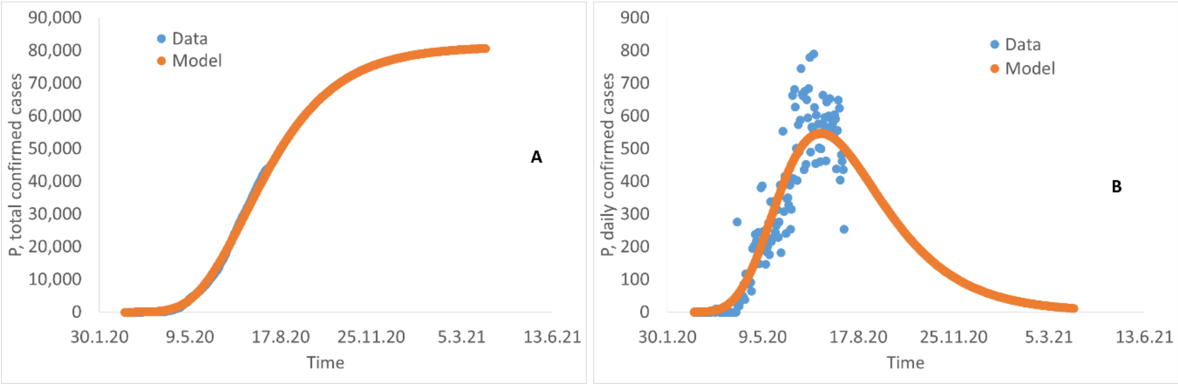


Figure 15: Model predictions of the full pandemic evolution of COVID-19 cases in Nigeria; A: Total confirmed cases B: Daily recorded cases. Data from NCDC (2020).

6. Conclusions

A model for describing the entire trajectory of the COVID-19 pandemic has been developed, demonstrating the exponential increase in the early part and decline resulting from various intervention programs. The balance equation modelling approach and SEIR modelling assumptions were employed in developing an intermediate resulting exponential model. In a similar pattern to the Arrhenius Equation, the model was modified with an inverse exponential term. The resulting model was able to describe the pandemic trend in China and Italy. It could also predict with precision COVID-19 data for Italy and Nigeria. Lastly, the model was used to predict the future of the pandemic in Nigeria if recent gains were sustained.

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Notations

Notation	Description	Units
a	exponential constant	$1/day$
B	Constant healthy population	$\#/m^2$
K	pseudo-exponential rate constant	$1/day$
K_0	disease infection rate constant	
k_F	pandemic spreading rate constant	m^2/day
P	Number density of confirmed cases	$\#/m^2$
P_H	Area density of the healthy population	$\#/m^2$
r_i	rate of generation of i people per unit area from various events	$\#/(m^2.t)$
t	Time	$days$
t_P	time for the peak daily increase	$days$

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A Comparative Analysis of the Shortest Path Algorithms for Covid-19 Activities And Distribution of Palliatives Items

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Abstract

In this research work, we examined the network of multiple sources to multiple destinations for COVID-19 activities and distribution of palliatives items. A systematic approach was adopted to derive the shortest route from given multiple sources to multiple destinations. In doing that, the adopted algorithm which was the modification of Dynamic programming problem to enable the relation solve multiple sources to multiple destinations problems directly without splitting such network problems into either: single source to single destination, multiple sources to single destination or single source to multiple destinations as has been the case with Dynamic programming algorithm, A search algorithm, Dijkstra's algorithm, Bellman-Ford-Moore algorithm, Floyd-Warshall algorithm, Johnson's algorithm and Veterbi algorithm. The adopted algorithm was used to solve the Nigeria road network problem of motorable roads from multiple sources in the coastal towns of Lagos, Warri, Port Harcourt and calabar to multiple destinations of border towns of Katsina, Sokoto and Maiduguri, and obtained the shortest possible routes for vehicular movement for COVID-19 activities and distribution of palliatives items. The adopted algorithm reduced the mathematical computations involved in the solution process and still obtained the optimal shortest path like other algorithms. The results of the research showed that, the adopted algorithm used less number of iterations, floating point arithmetic and less storage space which enhanced the speed of application of the method. The adopted algorithm had the ratio of networks used as 1 : M : MN for multiple sources to multiple destinations : single source to multiple destinations : single source to single destination.*

Keywords: *COVID-19* Road network, shortest route, multiple sources, multiple destinations, Covid-19 and Modified dynamic algorithm

1. Introduction

Covid-19 is a native of Asia initially originated in Wuhan, China, on 31 December 2019 where a cluster of Pneumonia cases were identified as a new type of novel coronavirus disease (Thompson, 2020) and the World Health Organization (WHO) named it Covid-19 on January 10, 2020 (WHO, 2020). On January 22, 2020, the People's Republic of China held a press conference and announced the prevention and control measures of the new coronavirus infection.

On the same day, the China's CDC introduced specimen collection, testing and diagnosing, tracking and management of close contacts, educative media enlightenment and risk management communication to the public (Page and Wei, 2020). All continents reported confirmed cases of Covid-19. Africa confirmed its first case in Egypt on Feb 14, 2020. China is Africa's leading commercial partner; thus, there are large travel volumes through which severe acute respiratory syndrome coronavirus could reach the continent. Several measures have already been implemented to prevent and control possible case importations from China and countries that are epicenter for the disease. However, the ability to limit and control local transmission after importation depends on the application and execution of strict measures of detection, prevention, and control. These measures include heightened surveillance and rapid identification of suspected cases, followed by patient transfer and isolation, rapid diagnosis, tracing, and follow-up of potential contacts (WHO, 2020). The application of such a vast technical and operational set of interventions depends on each country's public health and laboratory infrastructures and resources.

Nigeria is the most populous country in Africa with over 200 million citizens and the country's major cities were on lockdown since the index case. The Nigeria Centre for Disease Control (NCDC) reported the first confirmed Covid-19 case in Nigeria on February 27, 2020, when an Italian citizen in Lagos tested positive for the virus (Ehanire, 2020).

Covid-19 pandemic as an emergency public health challenge in the 21st Century has brought into limelight a need for interdisciplinary research. It has brought into picture the need for a trans-disciplinary view of the current crisis through various angles of global governance, technology and risk assessment (Zhang and Shaw, 2020).

Risk assessment or management has become essential tool during times of serious health crises, and McAleer (2020) emphasizes the importance of prevention relative to the cure. The outbreak and evolution of the Covid-19 pandemic has been analyzed from different perspectives. For example in the work of Hafner (2020): Li and Linton (2020) fit country-wise quadratic regressions to estimate the peak periods, Yue et al. (2020) analyze the impact of the pandemic on China's economy, Wang et al. (2020) consider the risk management of Covid-19 by universities in China, Yue et al. (2020) propose solutions and recommendations related to early warning, identification, and monitoring of risks, Liu et al. (2020) analyze the Chinese experience and its implications for other countries, Chang et al. (2020) present a charter for a sustainable travel, tourism, and hospitality industry for the time after Covid-19, Chang and McAleer (2020) critically evaluate the Global Health Security Index of 2019 which provides data before the discovery of Covid-19 and made it possible to evaluate how countries might have been prepared for a pandemic and acted accordingly. Further research analyzing the containment strategies of individual countries and global analysis of the Covid-19 situation includes Wang and Wang (2020), Zhao and Chen (2020), Han et al. (2020), and Di Gennaro et al. (2020).

This pandemic has disrupted activities all over the world and forced many countries to reset their economic priorities (Madubueze et al, 2020 and Iboi et al, 2020). Different control policies recommended by the WHO such as lockdown, quarantine, isolation, social distancing and restriction of the movement were implemented by the governments of many countries to halt the spread of COVID-19 (NCDC, 2020 and Okuonghae and Omame, 2020). Sadly, these control policies have caused economic down-turn, mortalities and morbidities, inflation, high rate of crime, lawlessness, and hunger in many countries (Okuonghae and Omame, 2020 and Ekundayo, 2020). The WHO and researchers have stated that COVID-19 will remain in the world until there is a vaccine (Scharping, 2020 and Schwartz, 2020). Since COVID-19 has come to stay in Nigeria

and life most also move on with the havoc created, there is need to optimize the little resources available with the best practices just like during and after the World War II, during which the use of operations research helped in fighting the war and the industrial revolution after the war respectively. Vehicular movement for activities of COVID-19 and distribution of palliatives items has become very necessary, for this to be achieved network analysis are needed.
Everything is a network, whenever we look at the interactions between things, a network is formed implicitly. Network of roads have become very important for road transporters, since it serves as a channel of moving people, goods and services from one place to another. In recent times, we have also observed that a lot of traffic congestion happens on our roads as a result of more influx of vehicles and poor state of some of the roads. Anyanwu *et al.* (1997) and Nwafor and Onya (2019) noted that, of all commodity movements to and from the sea-ports and airports, at least, two-thirds are handled by road transport while up to 90% of all other internal movement of goods and persons take place by roads. Olaniyi and Oniru (2017), also noted that, road transportation is demanded to execute the objectives of every other sectors in the economy. Not only does transportation provides mobility for people and goods, it helps shape an area's economic health and quality of life. Because of the high pertinence of road transportation, it is expedient for a country or nation to embark on integrating man, material, money and machinery towards the realization of diversified modes of transport before such country will boast to have achieved a diversified economy and a sustainable development. The Nigeria road network linked together all over the 36 states and the Federal Capital Territory with arterial roads which are interconnected with sub-arterial, distributor or collector roads and local roads which are full of challenges, particularly with pavement, although, this research work did not consider the challenges, rather the shortest route (optimal paths) from coaster towns through the several intermediate cities to border towns.
Nigeria has a national road network of about 200,000km; of this total, federal roads make up 18 per cent (about 36,000km), state roads 15 per cent (about 30,000km) and local government roads 67 per cent (about 134,000km), with most local government roads being unpaved. The road sector accounts for about 90 per cent of all freight and passenger movements in the country. Although the federal road network constitutes 18 per cent of the total national network, it accounts for about 70 per cent of the national vehicular and freight traffic. As at 2012, an estimated 40 per cent of the federal road network was in poor condition (in need of rehabilitation); 30 per cent in fair condition requiring periodic maintenance); and 27 per cent in good condition (requiring only routine maintenance). The remaining 3 per cent consists of unpaved trunk roads that need to be paved. In the case of state roads, 78 per cent is in poor condition, with 87 per cent of local government roads also considered to be in poor condition (Source: National Planning Commission, 2015).

It has been observed that other routes can be used for the same purpose of transporting people, goods and services from one location to the other, some of these roads can take a very long time which results in delay and even breakdown of vehicle, thus increasing the cost of maintaining such vehicles on the path of management. As a result, this research work seeks to obtain the shortest possible route for vehicular movement from multiple sources to multiple destinations through multiple-roads network system so as to minimize the cost of fueling, maintenance and loss of customers and get to their destinations in good time and also to compare the results of the existing algorithms with the adopted one.

2. Related works

Most traditional path finding solutions are based on shortest path algorithm that tend to minimize the cost of travel from point to another. Majority of work in network tomography have revolved on active problem from a single source. It has been noted that, the problem of identifying a multiple tomography amount to more than just matching nodes with the same label (Rabbat, 2004).

Networks have become very important for transporters, health sector, feeds formulation, security agencies, service providers, telecommunication, engineering and so many as Ahuja (1995) presented 42 applications of network drawn from the fields of operations research, computer science, physical sciences, medicine, engineering and applied mathematics, some of which are: Matrix rounding problem, Locating object in space, Urban traffic flows, Routing of multiple commodities, Local access telephone network, Multi-item production planning, and so on. Since, networks serve as a channel of connecting people, goods and services from place to another. One way to solve network problems is the shortest path problem. Shortest Path problems are one of the base operations of the network problems. The problem is to reach a target location(s) from a beginning location(s). There are generally numerous available paths to achieve this goal. The objective of the shortest path problem is to achieve this goal with the minimum distance or time traveled. The most traditional path finding solutions are based on shortest path algorithms that tend to minimize the cost of travel from point to another (Ehsan and Hunter, 2012). According to Zongxiang, (2013), multiple sources to multiple destination deals with many stops and attempts to achieve the shortest total distance. It plays an important role in saving resources and has application in a wide range of industries such as transportation, travel planning, production, agriculture, telecommunications, petroleum, education, military, road constructions, delivery services among others.

The shortest path problem is a problem of finding the minimum total distance or shortest path from a starting point to a final destination. Generally, in order to represent the shortest path problem we use graphs. A graph is a mathematical abstract object, which contain sets of vertices and edges. Edges connect pairs of vertices. Along the edges of a graph, it is possible to walk by moving from one vertex to other vertices depending on whether or not one can walk along the edges by both sides or by only one side determines if the graph is a directed graph or an undirected graph. In addition, lengths of edges are often called weights, and the weights are normally used for calculating the shortest path from one point to another (Kairanbay and Hajar, 2013). The shortest path of a network consisting of few nodes and arcs can easily be obtained with simple arithmetic. However, larger or complex networks can hardly be obtained with simple arithmetic, hence, the need for algorithm (Oladejo and Tamber, 2014). According to Wang et al. (2014), there are quite a number of existing shortest path search algorithms designed for shortest path problem. These include: Dynamic programming, A^* search, Dijkstra, Bellman-Ford-Moore, Floyd-Warshall, Johnson, Veterbi and linear programming.

According to Schrijver (2012), it is difficult to trace the history of the shortest path problem. One can imagine that, in very primitive societies (even animals), finding shortest paths (for instances, to food or water) is essential. He further added that, compared with other combinatorial optimization problem, like shortest spinning tree, assignment and transportation problem, the mathematical research in the shortest path problem started relatively late but Trueblood (1952) quoted that, “path problem were also studied at the beginning of 1950’s in the context of alternate path, that is, finding a second shortest route if the shortest path is blocked”. Zongxiang, (2013), further stated that, while the exact method exists, it is computationally very intensive, its computational complexity increases exponentially with the number of stops (destinations), which make such method practically infeasible in real world application. Zongxiang, also added that, the algorithm proposed in his work applied a divide-and-conquer rule. Betsikas (2005) stated that, for a multiple sources to multiple destinations network problem of shortest path to be solved, the network should be first converted to a single source to multiple destinations by adding a dummy node to the sources after which the A* algorithm can be applied, but for a dense (complicated) network like the case of our adopted network, its computational complexity increases exponentially or almost practically impossible to apply the method, hence need for a robust algorithm that can handle the network problem without conversion or tempering with the beauty of the original network.

The existing algorithm like: A* search, Dijkstra, Bellman-Ford-Moore, Floyd-Warshall, Johnson and Veterbi were used to determine the shortest route from single source to single destination, single source to multiple destinations, multiple sources to single destination (by reversing the order of the network to get single source to multiple destinations) and multiple sources to multiple destinations (but by splitting the entire network into individual single source to multiple destinations). However, the adopted algorithm was able to handle the four network models mentioned above and did not need the reversal of the network order nor splitting the entire network into individual single source to multiple destinations networks. This implies a reduction in the number of iterations required to solve a given network problem as compared with other existing algorithm, and hence a reduction in processing time and storage space (Sohana and Sazib, 2011). In the work of Wang et al (2012), a number of existing shortest path search algorithms were designed for the single pair shortest path problem includes those stated earlier. They added that, among the above mentioned algorithm, Dijkstra’s shortest path algorithm is the most commonly known to find the optimal shortest path from one origin to all vertices, while it can also be used to find the shortest path from one origin to one destination. Wang et al (2012) further noted that, A* search algorithm and its variants are the most popular path search algorithm designed for the shortest path problem implemented by major web-based map services to find the approximate shortest path between two nodes due to its low computational time and low memory consumption. It was also noted in Wang et al (2012) that, Dijkstra’s algorithm is the most generally utilized route finding algorithm for solving the shortest fasted and optimize path and others. They added that, Dijkstra’s algorithm is sometime called the single-source most limited way on the grounds that, it understands the single-source shortest path problem on a weighted guided chart ($G = V, E$), where G is the graph, V is a situation which component are called vertices (intersections, nodes or junctions) and E is the set of ordered pair of vertices called coordinated edges (roads, segments or arcs). Wang et al (2012) also noted that, Floyd’s algorithm is an example of dynamic programming algorithm which is used for solving shortest path problems. Rashid and Tariq (2018) added that, Floyd’s algorithm is used to find all paths from the source node to every other destination node in a network using matrix technique. They finally stated that Bellman-Ford algorithm is also one of the algorithms to find shortest path, but works for negative weights by: detecting a negative cycle if any exists, and finding shortest simple path if no negative cycle exists.

Many specialized algorithms and heuristics have been developed for combinatorial problems such as production scheduling and vehicle routing. The performance of the specialized algorithms is often sensitive to problem instances. As a common practice, researchers “tune” their algorithm to its best performance on the test set reported. This causes problems when the algorithms are to be used in industry applications where the algorithm configured for today's problem may perform poorly for tomorrow's instances. It is not practical to conduct massive

experiments on a regular basis for the selection of a more effective algorithm setting. The same situation applies when a number of alternative algorithms are available to the decision-maker for selection. The testing and comparison of heuristic algorithms has been a subject of much discussion in recent years. A familiar approach of algorithmic testing is to show, the adopted algorithm is better, at least in some aspect, than the current incumbent using either standard benchmark problems or randomly generated ones.
All the algorithms discussed above only solve network models of either single source to single destination or single source to multiple destinations but cannot handle the problem of multiple sources to multiple destinations directly except by splitting the problem into the form of either a single source to single destination, single vertex to all vertices or multiple sources to single destination (but with reversing the order before solving), after which the individual resulting networks are superimposed into its original form like in the work of Oladejo and Tamber (2014).

Sohana and Sazib (2011) noted that, the existing algorithms can be used to determine the shortest route between only two nodes (origin and destination) in a network. However, the adopted algorithm was able to handle the four network models of:

i. Single source to single destination;
ii. Single source to multiple destinations;
iii. Multiple sources to single destination;
iv. Multiple sources to multiple destinations.

Another advantage is that the adopted algorithm reduced the number of iterations taken to solve similar problem with other algorithms, this also implies a reduction in the time required to solve such problems as against earlier methods. The summary of some seven different algorithms as show in the table 1.

Table 1: The Seven Existing Algorithms

S/NO		DYNAMIC PROGRAMMING.	DIJSTRA'S	BELLMAN-FORD-MORE	ROY- FLOYD-WARSHALL	VITERBI	A* SEARCH	JOHNSON'S
1	**Year of Invention**	1949	1956	1958, 1956 and 1957	1959, 1962 and 1962	1967	1968	1977
2	**Inventors**	Richard Bellman	Edsger W. Dijkstra	Richard Bellman Lester Ford Jr Edward F. Moore	Bernard Roy Robert Floyd Stephen Warshall (Peter Ingerman (1962) nested the three authors)	Andrew Viterbi (though it has history of multiple invention Needleman and Wunsch & Wagner /Fischer)	Peter Hart Nils Nilsson Bertram Raphael	Donald B. Johnson
3	**Nature of weights**	Positive edge weight	Positive edge weight	Negative/positive weights but not negative cycle	Positive/negative edge weight	Positive edge weight	Positive weight	Negative weights not negative cycle
4	**Type of graph flow**	Directed	Directed (both)	Directed (both)	Both direct and undirected	Directed (forward)	Directed (both)	Directed (both)
5	**Network solution type**	Multiple sources to single destination and by reverser single source to multiple destination (call part SP)	Single source to multiple destination & by reverser multiple sources to single destination	Single source to multiple destination by reverse multiple source to single destination	Single source to multiple destination and by reverser multiple sources to single destination (SP to al vertices)	Multiple sources to single destination and by reverser to single destination all pair shortest path	Single source to multiple destination by reverse multiple sources to single destination	Single source to multiple destination and by reverser multiple sources to single destination
6	**Origin of the algorithm**	Original	Original	Original	Dynamic programming. Algorithm	Dynamic programming. Algorithm	Dijkstra's	Bellman ford F Dijkstra

7	**Application Areas**	-Road network - Resource allocation –invention management -speech recognition – geographical routing transpiration	Is usually the working principle behind link stage routing protocol – OSPF (Open shortest path first and –IS-IS (intermediate system to intermediate system) transportation	Distance vector routine protocols inform protocol transportation	-path finder networks - widest paths - maximum band width put - computing canonical form of difference bond matrices -optima routing etc	Speech recognition Speech synthesis Diarization Keyword spotting Computational linguistic Bioinformatics CDMA GSM digital cellular Dial up moderns satellite Deep space communication	Games, general graph traversal, transportation etc	Stochastic Process
8	**Computation time (Worst)**	O(E)	O(Elog(V))	O(VE)	$O(IVI^3)$	$O(VE^2)$	O((IEI))	$O(V^2logV+VE)$
9	**Recursive Function**	$v_n(S_n) = Min\{t_n(S_n) + v_{n-1}(S_{n-1}))\}, S_n = 1, 2, 3,\ldots, n-1$	$d_{ij}^{(m)} = min\{ d_{ij}^{(m-1)} + w_{kj}\}$	$v_n(S_n) = Min\{t_n(S_n) + v_{n-1}(S_{n-1}))\}, S_n = 1, 2, 3,\ldots, n-1$	Shortest Path $(i,j,k+1)$= min(shortest Path (i,j,k), shortest Path $(i,k+1,k)$ + shortest Path $(k+1,j,k)$)	$V_{1,k} = P(y_1 \mid k)\cdot\pi_k$ $V_{t,k} = \max_{x\in S}(P(y_t \mid k)\cdot a_{x,k}\cdot$	$f(n)=g(n)+h(n)$	$d_{ij}^{(t)}= min\{ d_{ij}^{(t-1)} + w_{kj}\}$

From the foregoing, this research work seeks to obtain the shortest possible route for vehicular movement from multiple sources to multiple destinations through multiple-roads network system for covid-19 activities and distribution of palliatives items and compare the result obtained from the existing algorithms with the proposed algorithm.

3. The Model

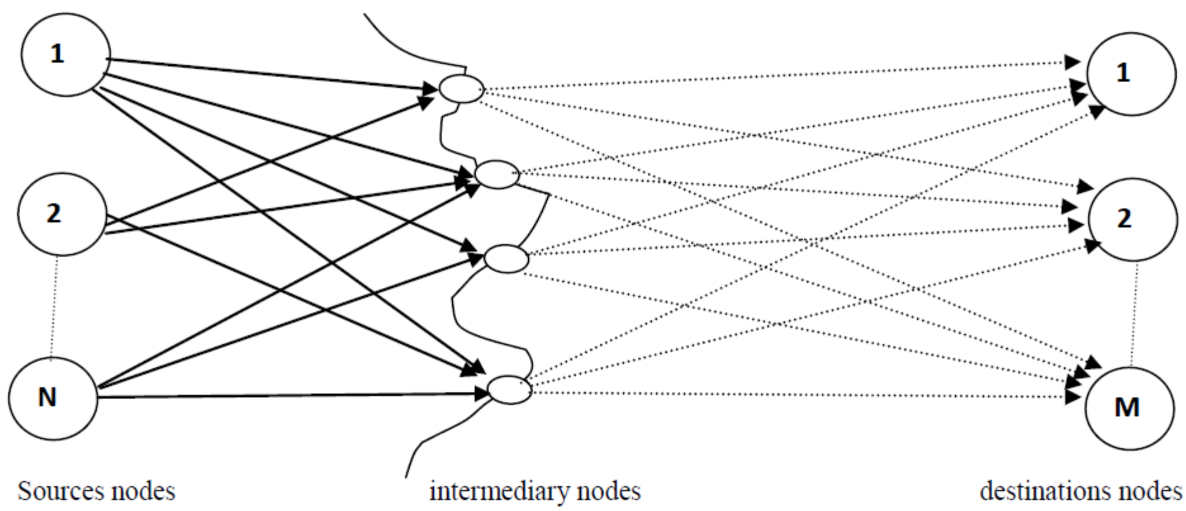

Figure 1: General form of multiple sources to multiple destinations network model.

The Nigeria road network model of multiple sources to multiple destinations for covid-19 activities and distribution of palliatives items described in this paper shown in Figure 2 was adopted from the work of Ikpotokin and Tamber (2017) which is the prototype of the general form of multiple sources to multiple destinations network model shown in Figure 1. The road network of motorable road from multiple sources (coastal towns: Lagos, Port-Harcourt, Asaba/warri and Calabar) to multiple destinations (border towns: Katsina, Sokoto and Maiduguri) passed through intermediate routes is shown in Figure 2. To find the shortest route from each of these sources to each of these destinations, by doing so, we adopted the modified version of the Dynamic programming algorithm proposed in Ikpotokin and Tamber (2017), which can solve the problem directly, with far less number of iterations, without splitting the network into single source to single destination or single source to multiple destinations. The ordinal numbers 1 to 75 in Figure 2 does not represent the distance, but the town number for ease of reference. The arrows in Figure 2 signify the direction of flow of possible route from the sources to the destinations. In order for all states (state capitals) to be reached, other states appear more than once, this is because the graph is a directed, therefore, the movement is only directed forward and for all states to be reached, we must model it this way. Therefore, the distance between each pair of identical state is 0, e.g. OSO to OSO = 0, IBD to IBD = 0,…, YOL to YOL = 0 .

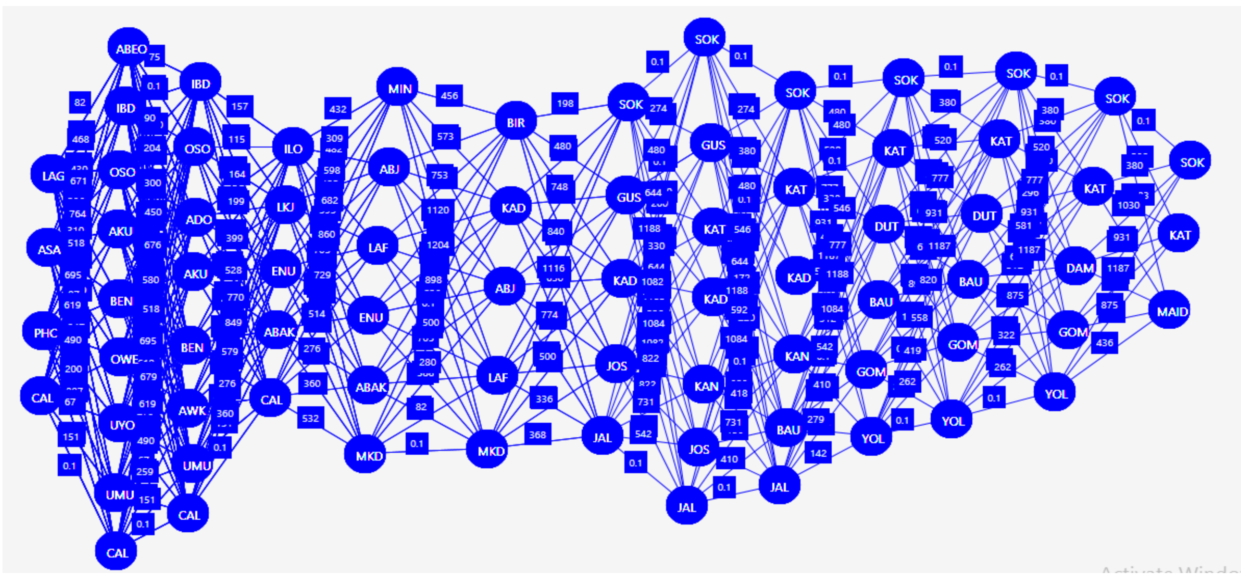

Figure 1: Nigeria road network of motorable road from multiple sources (coastal towns: Lagos, Port-Harcourt, Asaba/warri and Calabar) to multiple destinations (border towns: Katsina, Sokoto and Maiduguri).

4. Mathematical Formulation of the Problem

From the network model of multiple sources to multiple destinations of Nigeria road network adopted from Ikpotokin and Tamber (2017) in Figure 2, we modeled the following mathematical functions as a linear programming problem. According to Vishwanathan (2006) and Zainb et. al. (2019), formulating the linear programme (LP) for solving the shortest path from *s-t t,* we choose one variable per edge, x_e. If x_e is picked, $x_e = 1$ else $x_e = 0$ and w_e is the value of the weight of edge *e.* with the cost function:

$$\text{Minimize} \sum \sum_{(u,v)\ in\ network} W_{uv} X_{uv} \tag{1}$$

Subject to

$$\sum_{(k,v) in\ network} X_{kv} - \sum_{(u,k) in\ network} X_{uk} = \begin{cases} 1,\ if\ k = s(source) & (2) \\ 0\ for\ all\ other\ k & (3) \\ -1, if\ k = t\ (sink) & (4) \end{cases}$$

$0 \leq X_{uv} \leq 1$ and integer $\forall\ (u, v)$ in network (5)

Equation (2) implies that, the difference in the number of edges leaving source node *s* and entering into source node *s* is 1. Equation (3) implies also, that, for all other nodes, edges leaving them is equal the edges entering into them. Equation (4) means that the difference in the number of edges entering into destination node *t* and leaving destination node *t* is 1. Equation (5) is the non-negativity and integrality requirement.

Therefore, LP model of Nigeria road network of multiple sources to multiple destinations was developed by subjecting the data of Figure 2 into the general LP model of equation (1) to (5) the following models were generated:

Objective function:

Minimize

$$\begin{aligned}
& 82X_{1,5} + 132X_{1,6} + 237X_{1,7} + 341X_{1,8} + 322X_{1,9} + 564X_{1,10} + 689X_{1,11} + 600X_{1,12} + \\
& 772X_{1,13} + 468X_{2,5} + 430X_{2,6} + 396X_{2,7} + 310_{2,8} + 133X_{2,9} + 97X_{2,10} + 247X_{2,11} + \\
& 166X_{2,12} + 297X_{2,13} + 671X_{3,5} + 656X_{3,6} + 623X_{3,7} + 524X_{3,8} + 296X_{3,9} + 112X_{3,10} + \\
& 120X_{3,11} + 114X_{3,12} + 196X_{3,13} + 764X_{4,5} + 518X_{4,6} + 695X_{4,7} + 619X_{4,8} + 490X_{4,9} + \\
& 200X_{4,10} + 67X_{4,11} + 151X_{4,12} + 0X_{4,13} + 75X_{5,14} + 167X_{5,15} + 340X_{5,16} + 280X_{5,17} + \\
& 314X_{5,18} + 444_{5,19} + 595X_{5,20} + 764X_{5,21} + 0X_{6,14} + 90X_{6,15} + 264X_{6,16} + 204X_{6,17} + \\
& 300X_{6,18} + 457X_{6,19} + 580X_{6,20} + 518X_{6,21} + 90X_{7,14} + 0X_{7,15} + 178X_{7,16} + 118X_{7,17} + \\
& 259X_{7,18} + 425X_{7,19}\ 564X_{7,20} + 695X_{7,21} + 204X_{8,14} + 118X_{8,15} + 60X_{8,16} + 0X_{8,17} + \\
& 711X_{8,18} + 342X_{8,19} + 430X_{8,20} + 619X_{8,21} + 300X_{9,14} + 259X_{9,15} + 228X_{9,16} + 168X_{9,17} \\
& + 0X_{9,18} + 166X_{9,19} + 279X_{9,20} + 490X_{9,21} + 450X_{10,14} + 495X_{10,15} + 473X_{10,16} + \\
& 413X_{10,17} + 240X_{10,18} + 141X_{10,19} + 165X_{10,20} + 200X_{10,21} + 676X_{11,14} + 645X_{11,15} + \\
& 606X_{11,16} + 545X_{11,17} + 375X_{11,18} + 207X_{11,19} + 88X_{11,20} + 67X_{11,21} + 580X_{12,14} + \\
& 564X_{12,15} + 510X_{12,16} + 430X_{12,17} + 279X_{12,18} + 126X_{12,19} + 0X_{12,20} + 151X_{12,21} + \\
& 518X_{13,14} + 695X_{13,15} + 679X_{13,16} + 619X_{13,17} + 490X_{13,18} + 259X_{13,19} + 151X_{13,20} + \\
& 0X_{13,21} + 157X_{14,22} + 584X_{14,23} + 558X_{14,24} + 642X_{14,25} + 518X_{14,26} + 115X_{15,22} + \\
& 305X_{15,23} + 462X_{15,24} + 546X_{15,25} + 695X_{15,26} + 164X_{16,22} + 296X_{16,23} + 482X_{16,24} + \\
& 566X_{16,25} + 679X_{16,26} + 199X_{17,22} + 236X_{17,23} + 422X_{17,24} + 506X_{17,25} + 619X_{17,26} + \\
& 399X_{18,22} + 287X_{18,23} + 257X_{18,24} + 338X_{18,25} + 490X_{18,26} + 528X_{19,22} + 456X_{19,23} + \\
& 62X_{19,24} + 146X_{19,25} + 259X_{19,26} + 770X_{20,22} + 405X_{20,23} + 118X_{20,24} + 202X_{20,25} + \\
& 151X_{20,26} + 849X_{21,22} + 579X_{21,23} + 276X_{21,24} + 360X_{21,25} + 0X_{21,26} + 432X_{22,27} + \\
& 482X_{22,28} + 660X_{22,29} + 653X_{22,30} + 660X_{22,31} + 863X_{22,32} + 309X_{23,27} + 193\ X_{23,28} + \\
& 788\ X_{23,29} + 392\ X_{23,30} + 476\ X_{23,31} + 342\ X_{23,32} + 598X_{24,27} + 393\ X_{24,28} + 705\ X_{24,29} + \\
& 0\ X_{24,30} + 84\ X_{24,31} + 270\ X_{24,32} + 682X_{25,27} + 477X_{25,28} + 388X_{25,29} + 84X_{25,30} + 0X_{25,31} \\
& + 256X_{25,32} + 860X_{26,27} + 729\ X_{26,28} + 514\ X_{26,29} + 276\ X_{26,30} + 360\ X_{26,31} + 532\ X_{26,32} \\
& + 456X_{27,33} + 297\ X_{27,34} + 156\ X_{27,35} + 682X_{27,36} + 440X_{27,37} + 573X_{28,33} + 186X_{28,34} + \\
& 0X_{28,35} + 180X_{28,36} + 280X_{28,37} + 753X_{29,33} + 474X_{29,34} + 180X_{29,35} + 0X_{29,36} + 82X_{29,37} + \\
& 1120X_{30,33} + 772X_{30,34} + 393X_{30,35} + 705X_{30,36} + 270X_{30,37} + 1204X_{31,33} + 856X_{31,34} + \\
& 477X_{31,35} + 388X_{31,36} + 256X_{31,37} + 898X_{32,33} + 500X_{32,34} + 280X_{32,35} + 82X_{32,36} + 0X_{32,37} + \\
& 198X_{33,38} + 322X_{33,39} + 550X_{33,40} + 830X_{33,41} + 1266X_{33,42} + 480X_{34,38} + 260X_{34,39} +
\end{aligned}$$

$0X_{34,40}+\ 278X_{34,41}+\ 822X_{34,42}+\ 748X_{35,38}+\ 490X_{35,39}+\ 186X_{35,40}+\ 297X_{35,41}+$
$691X_{35,42}+\ 840X_{36,38}+\ 650X_{36,39}+\ 474X_{36,40}+\ 196X_{36,41}+\ 450X_{36,42}+\ 1116X_{37,38}+$
$774X_{37,39}+\ 500X_{37,40}+\ 336X_{37,41}+\ 368X_{37,42}+\ 0X_{38,43}+\ 274X_{38,44}\ +\ 380X_{38,45}+$
$480X_{38,46}+\ 546X_{38,47}+\ 644X_{38,48}+\ 1188X_{38,49}+\ 274X_{39,43}+\ 0X_{39,44}+\ 463X_{39,45}+$
$260X_{39,46}+\ 260X_{39,47}+\ 330X_{39,48}+\ 1082X_{39,49}+\ 480X_{40,43}+\ 260X_{40,44}+\ 428X_{40,45}+$
$0X_{40,46}+\ 220X_{40,47}+\ 278X_{40,48}+\ 822X_{40,49}+\ 644X_{41,43}+\ 330X_{41,44}+\ 592X_{41,45}+$
$278X_{41,46}+\ 418X_{41,47}+\ 0X_{41,48}+\ 542X_{41,49}+\ 1188X_{42,43}+\ 1082X_{42,44}+\ 1084X_{42,45}+$
$822X_{42,46}+\ 731X_{42,47}+\ 542X_{42,48}+\ 0X_{42,49}+\ 0X_{43,50}+\ 380X_{43,51}+\ 546X_{43,52}+\ 777X_{43,53}+$
$1188X_{43,54}+\ 1188X_{43,55}+\ 274X_{44,50}+\ 463X_{44,51}+\ 260X_{44,52}+\ 670X_{44,53}+\ 1082X_{44,54}+$
$1082X_{44,55}+\ 380X_{45,50}+\ 0X_{45,51}+\ 172X_{45,52}+\ 518X_{45,53}+\ 1084X_{45,54}+\ 1084X_{45,55}+$
$480X_{46,50}+\ 428X_{46,51}+\ 220X_{46,52}+\ 410X_{46,53}+\ 822X_{46,54}+\ 822X_{46,55}+\ 546X_{47,50}+$
$172X_{47,51}+\ 0X_{47,52}+\ 301X_{47,53}+\ 731X_{47,54}+\ 731X_{47,55}+\ 644X_{48,50}+\ 592X_{48,51}+$
$418X_{48,52}+\ 130X_{48,53}+\ 54X_{48,54}+\ 54X_{48,55}+\ 1188X_{49,50}+\ 1084X_{49,51}+\ 731X_{49,52}+$
$410X_{49,53}+ 0X_{49,54}+ 0X_{49,55} +0X_{50,56} + 480X_{50,57} + 520X_{50,58} + 777X_{50,59} + 931X_{50,60} +$
$1187X_{50,61} + 0X_{51,56} + 480X_{51,57} + 520X_{51,58} + 777X_{51,59} + 931X_{51,60} + 1187X_{51,61} +$
$480X_{52,56} + 0X_{52,57} + 338X_{52,58} + 410X_{52,59} + 566X_{52,60} + 875X_{52,61} + 546X_{53,56} +$
$172X_{53,57} + 136X_{53,58} + 301X_{53,59} + 457X_{53,60} + 714X_{53,61} + 777X_{54,56} + 516X_{54,57} +$
$312X_{54,58} + 0X_{54,59} + 155X_{54,60} + 416X_{54,61} + 1188X_{55,56} + 1084X_{55,57} + 542X_{55,58} +$
$410X_{55,59} + 279X_{55,60} + 142X_{55,61} + 0X_{56,62} + 380X_{56,63} + 520X_{56,64} + 777X_{56,65} +$
$931X_{56,66} + 1187X_{56,67} + 380X_{57,62} + 0\ X_{57,63} + 296X_{57,64} + 518X_{57,65} + 675X_{57,66} +$
$820X_{57,67} + 520X_{58,62} + 296X_{58,63} + 0X_{58,64} + 318X_{58,65} + 895X_{58,66} + 558X_{58,67} +$
$777X_{59,62} + 518X_{59,63} + 318X_{59,64} + 0X_{59,65} + 155X_{59,66} + 416X_{59,67} + 931X_{60,62}+$
$675X_{60,63} + 895X_{60,64} + 155X_{60,65} + 0\ X_{60,66}+ 262X_{60,67} + 1187X_{61,62} + 820X_{61,63} + 558$
$X_{61,64} + 416\ X_{61,65} + 262X_{61,66} + 0X_{61,67} + 0X_{62,68} + 380X_{62,69} + 1030X_{62,70} + 931X_{62,71}$
$+ 1187X_{62,72} + 380X_{63,68} + 0X_{63,69} + 603X_{63,70} + 671X_{63,71} + 875X_{63,72} + 520X_{64,68} +$
$296X_{64,69} + 288X_{64,70} + 895X_{64,71} + 558X_{64,72} + 777X_{65,68} + 581X_{65,69} + 312X_{65,70} +$
$155X_{65,71} + 416X_{65,72} + 931X_{66,68} + 675X_{66,69} + 256X_{66,70} + 0X_{66,71} + 262X_{66,72} +$
$1187X_{67,68} + 875X_{67,69} + 322X_{67,70} + 262X_{67,71} + 0X_{67,72} + 0X_{68,73} + 380X_{68,74} +$
$1123X_{68,75} + 380X_{69,73} + 0X_{69,74} + 794X_{69,75} + 1030X_{70,73} + 603X_{70,74} + 130X_{70,75} +$
$931X_{71,73} + 675X_{71,74} + 400X_{71,75} + 1187X_{72,73} + 875X_{72,74} + 436X_{72,75};$ (6)

Subject to: $X_{1,5} + X_{1,6} + X_{1,7} + X_{1,8} + X_{1,9} + X_{1,10} + X_{1,11} + X_{1,12} + X_{1,13} \leq 1;$ (7)

$X_{2,5} + X_{2,6} + X_{2,7} + X_{2,8} + X_{2,9} + X_{2,10} + X_{2,11} + X_{2,12} + X_{2,13} \leq 1;$ (8)

$X_{3,5} + X_{3,6} + X_{3,7} + X_{3,8} + X_{3,9} + X_{3,10} + X_{3,11} + X_{3,12} + X_{3,13} \leq 1;$ (9)

$X_{4,5} + X_{4,6} + X_{4,7} + X_{4,8} + X_{4,9} + X_{4,10} + X_{4,11} + X_{4,12} + X_{4,13} \leq 1;$ (10)

$X_{68,73} + X_{69,73} + X_{70,73} + X_{71,73} + X_{72,73} \geq 1;$ (11)

$X_{68,74} + X_{69,74} + X_{70,74} + X_{71,74} + X_{72,74} \geq 1;$ (12)

$X_{68,75} + X_{69,75} + X_{70,75} + X_{71,75} + X_{72,75} \geq 1;$ (13)

$X_{1,5} + X_{2,5} + X_{3,5}+ X_{4,5} =X_{5,14} + X_{5,15} + X_{5,16} + X_{5,17} + X_{5,18} + X_{5,19} +$

$X_{5,20} + X_{5,21} + X_{6,14};$ (14)

$$X_{1,6} + X_{2,6} + X_{3,6} + X_{4,6} = X_{6,15} + X_{6,16} + X_{6,17} + X_{6,18} + X_{6,19} + X_{6,20} + X_{6,21}; \quad (15)$$

$$X_{1,7} + X_{2,7} + X_{3,7} + X_{4,7} = X_{7,14} + X_{7,15} + X_{7,16} + X_{7,17} + X_{7,18} + X_{7,19} + X_{7,20} + X_{7,21}; \quad (16)$$

$$X_{1,8} + X_{2,8} + X_{3,8} + X_{4,8} = X_{8,14} + X_{8,15} + X_{8,16} + X_{8,17} + X_{8,18} + X_{8,19} + X_{8,20} + X_{8,21}; \quad (17)$$

$$X_{1,9} + X_{2,9} + X_{3,9} + X_{4,9} = X_{9,14} + X_{9,15} + X_{9,16} + X_{9,17} + X_{9,18} + X_{9,19} + X_{9,20} + X_{9,21}; \quad (18)$$

$$X_{1,10} + X_{2,10} + X_{3,10} + X_{4,10} = X_{10,14} + X_{10,15} + X_{10,16} + X_{10,17} + X_{10,18} + X_{10,19} + X_{10,20} + X_{10,21}; \quad (19)$$

$$X_{1,11} + X_{2,11} + X_{3,11} + X_{4,11} = X_{11,14} + X_{11,15} + X_{11,16} + X_{11,17} + X_{11,18} + X_{11,19} + X_{11,20} + X_{11,21}; \quad (20)$$

$$X_{1,12} + X_{2,12} + X_{3,12} + X_{4,12} = X_{12,14} + X_{12,15} + X_{12,16} + X_{12,17} + X_{12,18} + X_{12,19} + X_{12,20} + X_{12,21}; \quad (21)$$

$$X_{1,13} + X_{2,13} + X_{3,13} + X_{4,13} = X_{13,14} + X_{13,15} + X_{13,16} + X_{13,17} + X_{13,18} + X_{13,19} + X_{13,20} + X_{13,21}; \quad (22)$$

$$X_{5,14} + X_{6,14} + X_{7,14} + X_{8,14} + X_{9,14} + X_{10,14} + X_{11,14} + X_{12,14} + X_{13,14} = X_{14,22} + X_{14,23} + X_{14,24} + X_{14,25} + X_{14,26}; \quad (23)$$

$$X_{5,15} + X_{6,15} + X_{7,15} + X_{8,15} + X_{9,15} + X_{10,15} + X_{11,15} + X_{12,15} + X_{13,15} = X_{15,22} + X_{15,23} + X_{15,24} + X_{15,25} + X_{15,26}; \quad (24)$$

$$X_{5,16} + X_{6,16} + X_{7,16} + X_{8,16} + X_{9,16} + X_{10,16} + X_{11,16} + X_{12,16} + X_{13,16} = X_{16,22} + X_{16,23} + X_{16,24} + X_{16,25} + X_{16,26}; \quad (25)$$

$$X_{5,17} + X_{6,17} + X_{7,17} + X_{8,17} + X_{9,17} + X_{10,17} + X_{11,17} + X_{12,17} + X_{13,17} = X_{17,22} + X_{17,23} + X_{17,24} + X_{17,25} + X_{17,26}; \quad (26)$$

$$X_{5,19} + X_{6,19} + X_{7,19} + X_{8,19} + X_{9,19} + X_{10,19} + X_{11,19} + X_{12,19} + X_{13,19} = X_{19,22} + X_{19,23} + X_{19,24} + X_{19,25} + X_{19,26}; \quad (27)$$

$$X_{5,20} + X_{6,20} + X_{7,20} + X_{8,20} + X_{9,20} + X_{10,20} + X_{11,20} + X_{12,20} + X_{13,20} = X_{20,22} + X_{20,23} + X_{20,24} + X_{20,25} + X_{20,26}; \quad (28)$$

$X_{5,21} + X_{6,21}X_{7,21} + X_{8,21} + X_{9,21} + X_{10,21} + X_{11,21} + X_{12,21} + X_{13,21} =$

$X_{21,22} + X_{21,23} + X_{21,24} + X_{21,25} + X_{21,26};$ (29)

$X_{14,22} + X_{15,22} + X_{16,22} + X_{17,22} + X_{18,22} + X_{19,22} + X_{20,22} + X_{21,22} =$

$X_{22,27} + X_{22,28} + X_{22,29} + X_{22,30} + X_{22,31} + X_{22,32};$ (30)

$X_{14,23} + X_{15,23} + X_{16,23} + X_{17,23} + X_{18,23} + X_{19,23} + X_{20,23} + X_{21,23}=$

$X_{23,27} + X_{23,28} + X_{23,29} + X_{23,30} + X_{23,31} + X_{23,32};$ (31)

$X_{14,24} + X_{15,24} + X_{16,24} + X_{17,24} + X_{18,24} + X_{19,24} + X_{20,24} + X_{21,24} =$

$X_{24,27} + X_{24,28} + X_{24,29} + X_{24,30} + X_{24,31} + X_{24,32};$ (32)

$X_{14,25} + X_{15,25} + X_{16,25} + X_{17,25} + X_{18,25} + X_{19,25} + X_{20,25} + X_{21,25} =$

$X_{25,27} + X_{25,28} + X_{25,29} + X_{25,30} + X_{25,31} + X_{25,32};$ (33)

$X_{14,26} + X_{15,26} + X_{16,26}X_{17,26} + X_{18,26} + X_{19,26} + X_{20,26} + X_{21,26} =$

$X_{26,27} + X_{26,28} + X_{26,29} + X_{26,30} + X_{26,31} + X_{26,32};$ (34)

$X_{22,27} + X_{23,27} + X_{24,27} + X_{25,27} + X_{26,27} = X_{27,33} + X_{27,34} + X_{27,35} +$

$X_{27,36} + X_{27,37};$ (35)

$X_{22,28} + X_{23,28} + X_{24,28} + X_{25,28} + X_{26,28} = X_{28,33} + X_{28,34} + X_{28,35} +$

$X_{28,36} + X_{28,37};$ (36)

$X_{22,29} + X_{23,29} + X_{24,29} + X_{25,29} + X_{26,29} = X_{29,33} + X_{29,34} + X_{29,35} +$

$X_{29,36} + X_{29,37};$ (37)

$X_{22,30} + X_{23,30} + X_{24,30} + X_{25,30} + X_{26,30} = X_{30,33} + X_{30,34} + X_{30,35} +$

$X_{30,36} + X_{30,37};$ (38)

$X_{22,31} + X_{23,31} + X_{24,31} + X_{25,31} + X_{26,31} = X_{31,33} + X_{31,34} + X_{31,35} +$

$X_{31,36} + X_{31,37};$ (39)

$X_{22,32} + X_{23,32} + X_{24,32} + X_{25,32} + X_{26,32} = X_{32,33} + X_{32,34} + X_{32,35} +$

$X_{32,36} + X_{32,37};$ (40)

$X_{27,33} + X_{28,33} + X_{29,33} + X_{30,33} + X_{31,33} + X_{32,33} = X_{33,38} + X_{33,39} + X_{33,40} +$

$X_{33,41} + X_{33,42};$ (41)

$X_{27,34} + X_{28,34} + X_{29,34} + X_{30,34} + X_{31,34} + X_{32,34} = X_{34,38} + X_{34,39} + X_{34,40} +$

$X_{34,41}+ X_{34,42};$ (42)

$X_{27,35} + X_{28,35}+ X_{29,35}+ X_{30,35} + X_{31,35}+ X_{32,35} = X_{35,38}+ X_{35,39}+ X_{35,40}$

$+ X_{35,41}+ X_{35,42};$ (43)

$X_{27,36}+ X_{28,36}+ X_{29,36}+ X_{30,36}+ X_{31,36}+ X_{32,36} = X_{36,38}+ X_{36,39}+ X_{36,40}$

$+ X_{36,41}+ X_{36,42};$ (44)

$X_{27,37}+ X_{28,37} + X_{29,37}+ X_{30,37}+ X_{31,37}+ X_{32,37} = X_{37,38}+ X_{37,39}+ X_{37,40}$

$+ X_{37,41}+ X_{37,42};$ (45)

$X_{33,38}+ X_{34,38}+X_{35,38}+X_{36,38}+X_{37,38} = X_{38,43}+ X_{38,44} + X_{38,45}+ X_{38,46}$

$+ X_{38,47}+ X_{38,48}+ X_{38,49};$ (46)

$X_{33,39}+X_{34,39}+X_{35,39}+X_{36,39}+X_{37,39} = X_{39,43}+ X_{39,44}+ X_{39,45}+ X_{39,46}$

$+ X_{39,47}+ X_{39,48}+ X_{39,49};$ (47)

$X_{33,40}+ X_{34,40}+X_{35,40}+X_{36,40}+X_{37,40} = X_{40,43}+ X_{40,44}+ X_{40,45}+ X_{40,46}$

$+ X_{40,47}+ X_{40,48}+ X_{40,49};$ (48)

$X_{33,41}+ X_{34,41}+X_{35,41}+X_{36,41}+X_{37,41} = X_{41,43}+ X_{41,44}+ X_{41,45}+ X_{41,46}$

$+ X_{41,47}+ X_{41,48}+ X_{41,49};$ (49)

$X_{33,42}+ X_{34,42}+X_{35,42}+X_{36,42}+X_{37,42} = X_{42,43}+ X_{42,44}+ X_{42,45}+ X_{42,46}$

$+ X_{42,47}+ X_{42,48}+ X_{42,49};$ (50)

$X_{38,43}+X_{39,43}+X_{40,43}+X_{41,43}+X_{42,43}= X_{43,50}+ X_{43,51}+ X_{43,52}+ X_{43,53}$

$+ X_{43,54}+ X_{43,55};$ (51)

$X_{38,44} + X_{39,44}+X_{40,44}+X_{41,44}+X_{42,44}= X_{44,50}+ X_{44,51}+ X_{44,52}+ X_{44,53}$

$+ X_{44,54}+ X_{44,55};$ (52)

$X_{38,45}+ X_{39,45}+X_{40,45}+X_{41,45}+X_{42,45}= X_{45,50}+ X_{45,51}+ X_{45,52}+X_{45,53}$

$+ X_{45,54}+ X_{45,55};$ (53)

$X_{38,46}+X_{39,46}+X_{40,46}+X_{41,46}+X_{42,46}= X_{46,50}+ X_{46,51}+ X_{46,52}+ X_{46,53}+$

$X_{46,54}+ X_{46,55};$ (54)

$X_{38,47}+ X_{39,47}+X_{40,47}+X_{41,47}+X_{42,47}= X_{47,50}+ X_{47,51}+ X_{47,52}+ X_{47,53}+$

$X_{47,54}+ X_{47,55};$ (55)

$X_{38,48}+ X_{39,48}+X_{40,48}+X_{41,48}+X_{42,48}= X_{48,50}+ X_{48,51}+ X_{48,52}+ X_{48,53}+$

$X_{48,54}+ X_{48,55};$ (56)

$X_{38,49}+X_{39,49}+X_{40,49}+X_{41,49}+X_{42,4} = X_{49,50}+ X_{49,51}+ X_{49,52}+ X_{49,53}+$

$X_{49,54}+ X_{49,55};$ (57)

$X_{43,50}+ X_{44,50}+X_{45,50}+X_{46,50}+X_{47,50}+X_{48,50}+X_{49,50}= X_{50,56} + X_{50,57} +$

$X_{50,58} + X_{50,59} + X_{50,60} + X_{50,61};$ (58)

$X_{43,51}+ X_{44,51}+X_{45,51}+X_{46,51}+X_{47,51}+X_{48,51}+X_{49,51}= X_{51,56} + X_{51,57} +$

$X_{51,58} + X_{51,59} + X_{51,60} + X_{51,61};$ (59)

$X_{43,52}+ X_{44,52}+X_{45,52}+X_{46,52}+X_{47,52}+X_{48,52}+X_{49,52}= X_{52,56} + X_{52,57} +$

$X_{52,58} + X_{52,59} + X_{52,60} + X_{52,61};$ (60)

$X_{43,53}+ X_{44,53}+X_{45,53}+X_{46,53}+X_{47,53}+X_{48,53}+X_{49,53}= X_{53,56} + X_{53,57} +$

$X_{53,58} + X_{53,59} + X_{53,60} + 714X_{53,61};$ (61)

$X_{43,54}+ X_{44,54}+X_{45,54}+X_{46,54}+X_{47,54}+X_{48,54}+X_{49,54}= X_{54,56} + X_{54,57} +$

$X_{54,58} + X_{54,59} + X_{54,60} + X_{54,61};$ (62)

$X_{43,55}+ X_{44,55}+X_{45,55}+X_{46,55}+X_{47,55}+X_{48,55}+X_{49,55} =X_{55,56} + X_{55,57} +$

$X_{55,58} + X_{55,59} + X_{55,60} + X_{55,61};$ (63)

$X_{51,56} + X_{52,56} +X_{53,56} +X_{54,56} +X_{55,56} = X_{56,62} + X_{56,63} + X_{56,64} +$

$X_{56,65} + X_{56,66} + X_{56,67};$ (64)

$X_{51,57} + X_{52,57} +X_{53,57} +X_{54,57} +X_{55,57} = X_{57,62} + X_{57,63} + X_{57,64} +$

$X_{57,65} + X_{57,66} + X_{57,67};$ (65)

$X_{51,58} +X_{52,58} +X_{53,58} +X_{54,58} +X_{55,58} = X_{58,62} + X_{58,63} + X_{58,64} +$

$X_{58,65} + X_{58,66} + X_{58,67};$ (66)

$X_{51,59} + X_{52,59} +X_{53,59} +X_{54,59} +X_{55,59} = X_{59,62} + X_{59,63} + X_{59,64} +$

$X_{59,65} + X_{59,66} + X_{59,67};$ (67)

$X_{51,60} + X_{52,60} +X_{53,60} +X_{54,60} +X_{55,60} = X_{60,62}+ X_{60,63} + X_{60,64} +$

$X_{60,65} + X_{60,66}+ X_{60,67};$ (68)

$X_{51,61} + X_{52,61} +X_{53,61} +X_{54,61} +X_{55,61} = X_{61,62} + X_{61,63} + X_{61,64} +$

$X_{61,65} + X_{61,66} + X_{61,67};$ (69)

$X_{56,62} + X_{57,62} + X_{58,62} + X_{59,62} + X_{60,62} + X_{61,62} = X_{62,68} + X_{62,69} +$

$X_{62,70} + X_{62,71} + X_{62,72};$ (70)

$X_{56,63} + X_{57,63} + X_{58,63} + X_{59,63} + X_{60,63} + X_{61,63} = X_{63,68} + X_{63,69} +$

$X_{63,70} + X_{63,71} + X_{63,72};$ (71)

$X_{56,64} + X_{57,64} + X_{58,64} + X_{59,64} + X_{60,64} + X_{61,64} = X_{64,68} + X_{64,69} +$

$X_{64,70} + X_{64,71} + X_{64,72};$ (72)

$X_{56,65} + X_{57,65} + X_{58,65} + X_{59,65} + X_{60,65} + X_{61,65} = X_{65,68} + X_{65,69} +$

$X_{65,70} + X_{65,71} + X_{65,72};$ (73)

$X_{56,66} + X_{57,66} + X_{58,66} + X_{59,66} + X_{60,66} + X_{61,66} = X_{66,68} + X_{66,69} +$

$X_{66,70} + X_{66,71} + X_{66,72};$ (74)

$X_{56,67} + X_{57,67} + X_{58,67} + X_{59,67} + X_{60,67} + X_{61,67} = X_{67,68} + X_{67,69} +$

$X_{67,70} + X_{67,71} + X_{67,72};$ (76)

$X_{62,68} + X_{63,68} + X_{64,68} + X_{65,68} + X_{66,68} + X_{67,68} = X_{68,73} + X_{68,74} + X_{68,75};$ (77)

$X_{62,69} + X_{63,69} + X_{64,69} + X_{65,69} + X_{66,69} + X_{67,69} = X_{69,73} + X_{69,74} + X_{69,75};$ (78)

$X_{62,70} + X_{63,70} + X_{64,70} + X_{65,70} + X_{66,70} + X_{67,70} = X_{70,73} + X_{70,74} + X_{70,75};$ (79)

$X_{62,71} + X_{63,71} + X_{64,71} + X_{65,71} X_{66,71} + X_{67,71} = X_{71,73} + X_{71,74} + X_{71,75};$ (80)

$X_{62,72} + X_{63,72} + X_{64,72} + X_{65,72} + X_{66,72} + X_{67,72} = X_{72,73} + X_{72,74} + X_{72,75};$ (81)

$X_{i,j} \geq 0$, $i = 1, 2, 3, \ldots 72$ and $j = 5, 6, \quad 7, \ldots 75$ (82)

5.0 The Adopted Algorithm

The generalized dynamic programming method for solving complex problems by breaking it down into a collection of simpler sub-problems, solving each of the sub-problems just once, and storing their solutions ideally, using a memory-based data structure was applied. Bellman described equation (83) as the functional recursive equation for the shortest path problem:

$V_n(S_n) = Min\ \{t_n(S_n) + V_{n-1}(S_{n-1}))\}, S_n = 1, 2, 3, \ldots, n-1$ (83)

Where: $V_n(S_n)$ = optimal value over the current state and subsequent stages, given that we are in state S_n with n stages to go. $t_n(S_n)$ = the distance in state S_n with n stages to go. $V_{n-1}(S_{n-1})$ = optimal value before the current state and previous stages, given that we are in state S_n with n

stages to go. S_n = number of stages n = number of states $V_0(S_0) = t_0(S_0)$, $S_0 = 1, 2, 3, \ldots, N$ Immediate decision is S_{n-1}, and optimal decisions are made thereafter (Wang et al, 2012).

The generalized dynamic programming method cannot solve multiple sources to multiple destinations problem directly. It can either determine the first destination in the multiple sources of multiple destinations problem or it can solve the multiple sources to multiple destinations problem by decomposing it into single source to single destination then solve these networks individually before superimposing the results to one network. However, the proposed algorithm which is a modification in the procedure of applying the Bellman generalized algorithm can solve multiple sources to multiple destination problems and obtain the shortest route from different sources to the different destinations. However, we discovered that the algorithm can also solve the other three categories of:

I. Single source to single destination network problem
II. Single source to multiple destination network problem (without reversing the network problem order) and
III. Multiple sources to single destination network problem (without reversing the network problem order).

5.1 The Procedure of the Adopted Algorithm

The following are the steps of the adopted Algorithm:

Step 1: Identify the problem decision variables and specify the objective function to be optimized under certain conditions, (if any)

Step 2: Decompose (or divide) the given network problem into a number of smaller sub-problems (or stages). Identify the state variables at each stage and write down the transformation function as a function of the state variables and decision variables at the next stage.

Step 3: If the last stage has more than one destination (state), decompose (or divide) the last stage, into individual destination (state), otherwise, proceed to step 4.

Step 4: Write down a general recursive relationship for computing the optimal policy. Decide whether to follow the forward or the backward method for solving the problem.

Step 5: Construct appropriate tables to show the required values of the return function at each stage as shown in the table 1 below.

Step 6: Determine the overall optimal policy or decisions and its value at each stage. There may be more than one such optimal policy.

6.0 Results

Subjecting the data of Figure 2 which represent the distances from town(s) to the others in the network and equations (6) to (82) to the adopted algorithm and LINGO a computer package respectively, since, it was difficult to solve the problem (LP) manually. The following results were obtained showing the shortest path through the network. The results are also depicted in Figure 3 below:

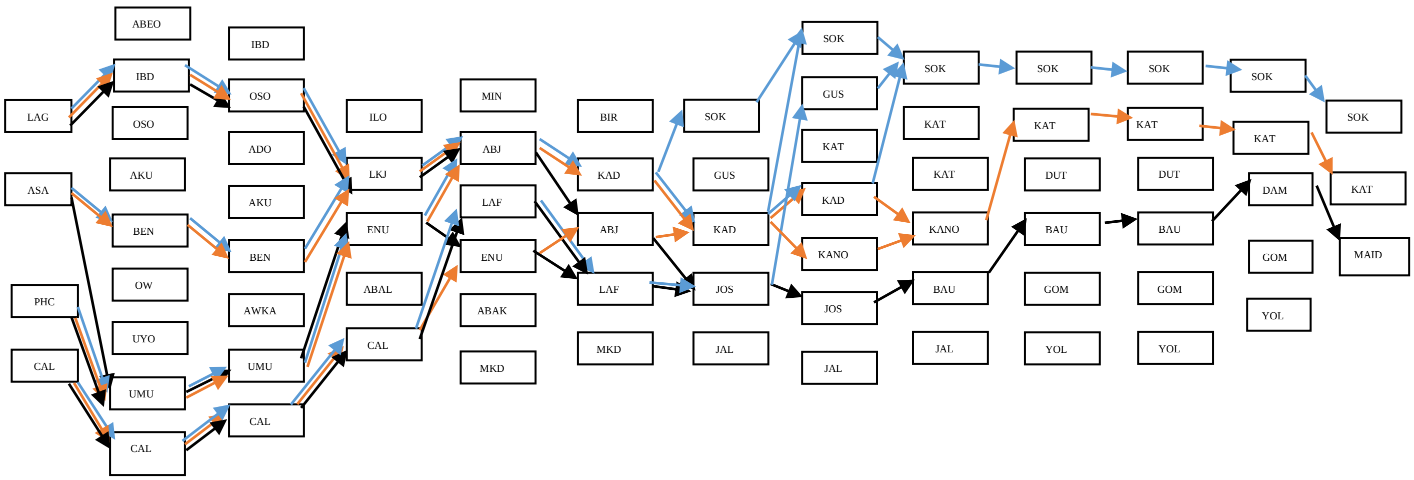

Figure 3: Optimal path of Nigeria road network of motorable road from multiple sources (coastal towns: Lagos, Port-Harcourt, Asaba/warri and Calabar) to multiple destinations (border towns: Katsina, Sokoto and Maiduguri)

From figure 3, the shortest paths of Nigeria road network of motorable road from multiple sources (coastal towns: Lagos, Port-Harcourt, Asaba/warri and Calabar) to multiple destinations (border towns: Katsina, Sokoto and Maiduguri) for COVID-19 activities and distribution of palliatives items were obtained as follow:

1) a. Lagos to Sokoto: Lagos-Ibadan-Oshogbo-Lokoja-Abuja-Kaduna-Sokoto = 1375km

b. Lagos to Katsina: Lagos- Ibadan-Oshogbo-Lokoja-Abuja-Kaduna-Kano-Katsina =1298km

c. Lagos to Maiduguri: Lagos- Ibadan-Oshogbo-Lokoja-Abuja-Jos-Bauchi-Damaturu-Maiduguri =1589km

2) a. Asaba to Sokoto: Asaba-Benin-Lokoja-Abuja-Kaduna-Sokoto =1279km

b. Asaba to Katsina: Asaba-Benin-Lokoja-Abuja-Kaduna-Kano-Katsina =1191

c. Asaba to Maiduguri: Asaba-Umuahia-Enugu-Lafia-Jos- Bauchi-Damaturu-Maiduguri =1404km

3) a. Port Harcourt to Sokoto: PHC-Umuahia-Enugu- Abuja-Kaduna-Sokoto =1291km

b. Port Harcourt to Katsina: PHC-Umuahia-Enugu- Abuja-Kaduna-Kano-Katsina =1203km

c. Port Harcourt to Maiduguri: PHC-Umuahia-Enugu- Lafia-Jos- Bauchi-Damaturu-Maiduguri =1352km

4) a. Calabar to Sokoto: Calabar-Lafia-Jos-Gusau-Sokoto = 1314km

b. Calabar to Katsina: Calabar-Enugu- Abuja-Kaduna-Kano-Katsina =1240km

c. Calabar to Maiduguri: Calabar-Lafia-Jos- Bauchi-Damaturu-Maiduguri =1282km.

Although, the results obtained using the existing algorithms were the same with the adopted algorithm, the number of iterations, number of individual networks and computational complexity were reduced using the adopted algorithm as summarized in table 3.

Table 3: A Comparative Analysis of the Existing Algorithms with the Adopted Algorithm

	A* search	Bellman – Ford-Moore	Dijkstra	Floyd-Warshall	Johnson	Viterbi	Dynamic proramming.	Modified Dynamic proramming.
Single source to single destination of our model (a) number of network	12	12	12	12	12	12	12	12
(b) number of vertices number of edges	71 390 } X12	71 390 } 12	71 390 } X12	71 390 } X12	71 390 } X12	71 390 } X12	71 390 } X12	71 390 } X12
Computational number (Floating points)	$1.1086x10^{185}$ O(/E^{v}/)x12	332,280 O(VE)x12	10241.1612 O(|V|+|E|)log (V)x12	4,294,932 O(/V/^{3})x12	444,266.32 O(V^{2})logv+VE)x12	129589200 O(VE^{2})x12	4680 O(E)x12	**4680 O(E)x12**
Single source to multiple destination of our mode (a) number of network	4	4	4	4	4	4	4	4
(b) i. No of vertices ii. No of edges	73 400 } X4	73 400 } X4	73 400 } X4	73 400 } X4	73 400 } X4	73 400 } X4	73 400 } X4	73 400 } X4
Computational number (floating point)	$3.5681x10^{190}$ O(/E^{v}/)x4	116,800 O(VE)x4	3525.40685 O(|V|+|E|)log (V)x4	1,556,068 O(/V/^{3})x4	156518.59 O(V^{2}logv+VE)x4	46,720,000 O(VE^{2})x4	1600 O(E)x4	1600 O(E)x4
Multiple sources to single destination of our model No of network:	3 (but can only work by reversing the network)	3 (but can only work by reversing the network)	3 (but can only work by reversing the network)	3 (but can only work by reversing the network)	3 (but can only work by reversing the network)	3 (but can only work by reversing the network)	3	3
i) No of vertices ii) No of edges	73 417 } X3	73 417 } X3	73 417 } X3	73 417 } X3	73 417 } X3	73 417 } X3	73 417 } X3	73 417 } X3
Computational number (floating point)	$5.5854x10^{191}$ O(/E^{v}/)x3	92,574 O(VE)x3	2,739.0846 O(|V|+|E|)log (V)x3	1,215,672 O(/V/^{3})x3	123281.74 O((V^{2})LogV+VE)x3	38,603,358 O(VE^{2})x3	1251 O(E)x3	1251 O(E)x3
Multiple sources to multiple destinations of our model	Cannot work directly without modification	Cannot work directly without modification	Cannot work directly without modification	Cannot work directly without modification	Cannot work directly without modification	Cannot work directly without modification	Cannot work directly without modification	Can work directly without modification

No of network:	1	1	1	1	1	1	1	1
i) vertices ii)f edges	75 417	75 417	75 417	75 417	75 417	75 417	75 417	75 427
Computational number (floating point)	∞ $O(/E^v/)$	∞ O(VE)	∞ O(|V|+|E|)log (V)x3	∞ $O(/V/^3)$	∞ $O((V^2)LogV+VE)$	∞ $O(VE^2)$	∞ O(VE)	427 O(E)

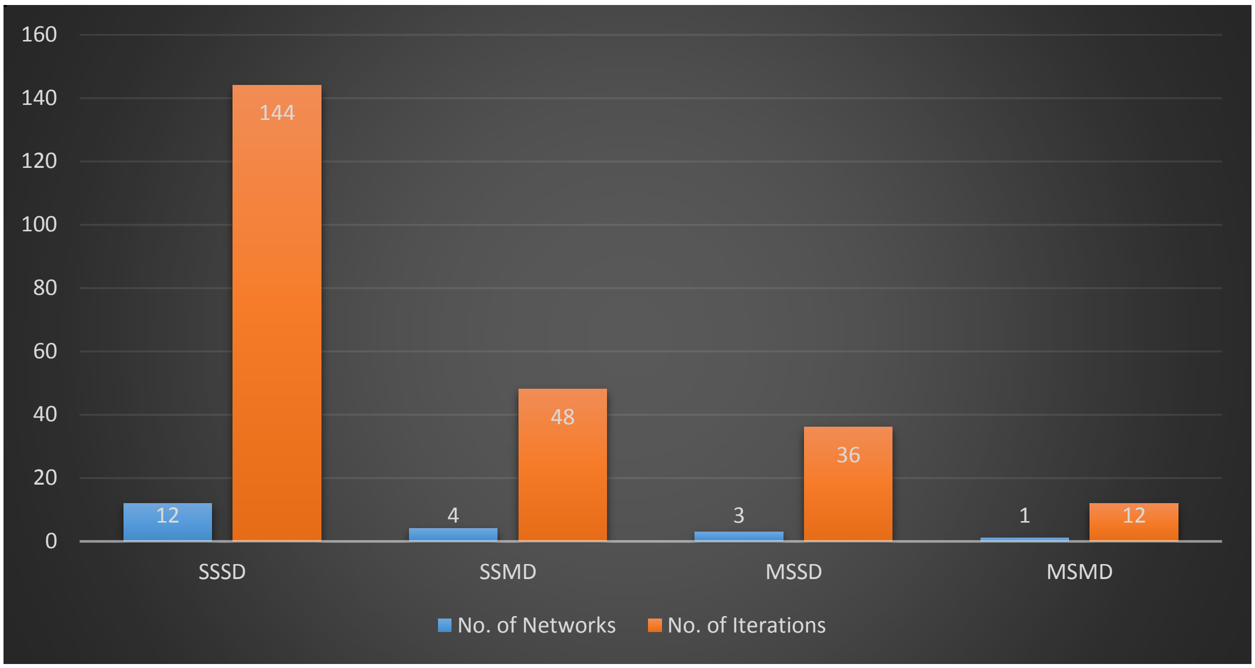

Figure 4: Number of networks using the four networks types solution.

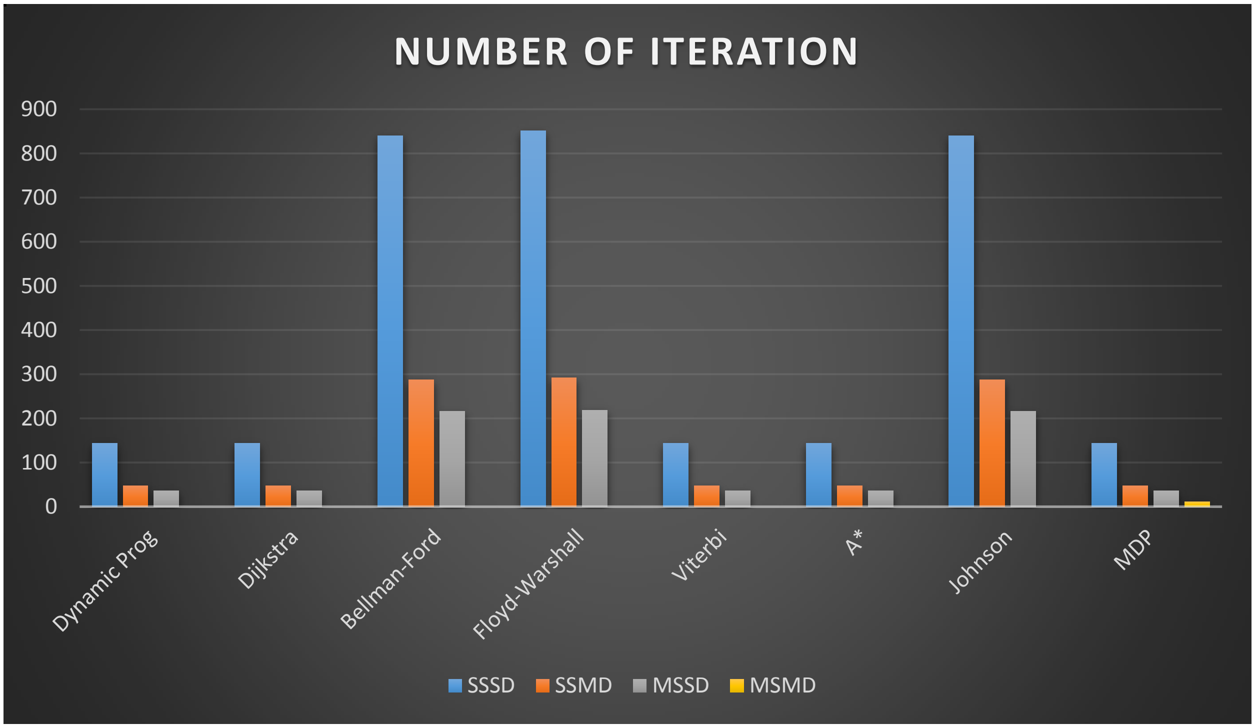

Figure 5: Number of iterations using the solution of the eight algorithms.

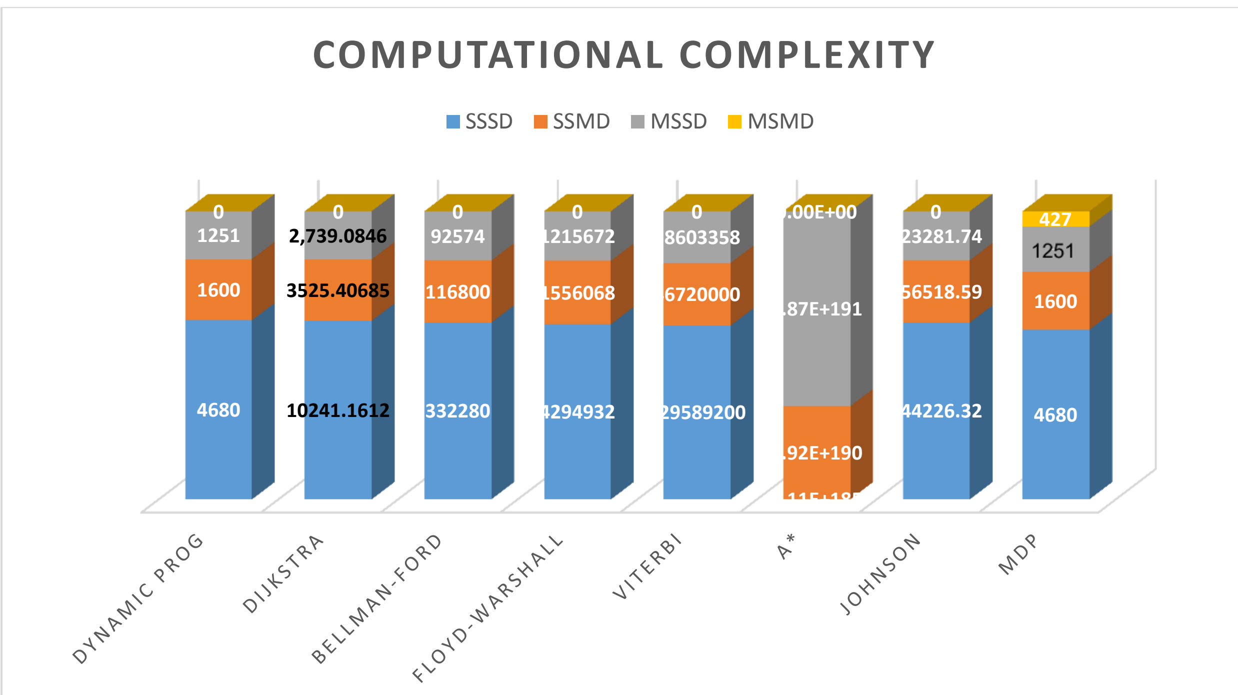

Figure 6: Computational complexity of the eight algorithms.

7.0 Discussion

From the foregoing, the method of single source to single destination used 12 networks with 144 iterations, single source to multiple destinations used 4 networks with 48 iterations. In the reversed order the multiple sources to single destination used 3 networks with 36 iterations. Whereas, our proposed method, used only one (1) network model with twelve (12) iterations to obtain the same optimal solution. In terms of number of iterations, with one model being used in the adopted method as against the 4 and 12 network models of single source-multiple destinations and single source-single destination respectively, we have a ratio of *12:48:144 => 1:4:12.*

Similarly, in terms of networks used in solving the problem, we have a ratio of *1:4:12* of our proposed method : single source-multiple destinations : single source-single destination as shown in Figure 4.

From the foregoing, there appear to be a relationship between the number of networks and number of iterations. Hence, in general, if we have *M-sources* and *N-destinations* where the network is divided into *S-stages*, then, the number of iterations with one model been used as in our proposed method compare to the M and MN models of single source-multiple destinations and single source-single destination respectively, we have a ratio of *1*S : M*S : M*N*S* and the ratio of networks used is *1 : M : MN*. This shows that, the number of iterations used is the number of stages *(S)* multiplied by the network splits of the network model that is *S(1 : M : MN)* for multiple sources to multiple destinations : single source to multiple destinations : single source to single destination.

Also in this research work, the model of Nigeria road network from multiple sources to multiple destinations through several intermediary nodes (states) was adopted from the work of Ikpotokin and Tamber (2017) with its linear programming problem formulated with 441 variables and 75 constraints. Due to the large number of variables and constraints, it was difficult to solve this manually. Hence we employed the use of LINGO computer application package which solves any LPP directly.

In addition, all the existing methods applied 12 networks for single source-single destination, 4 for single source-multiple destinations and 3 for multiple sources-single destination whereas, the adopted method, has only one network as also depicted in Figure 5. In the single source-single destination, the number of vertices and edges are 840 and 4680 respectively when all the existing algorithms were used. Further, the number of iterations used by 5 out of 8 methods (including ours) is 144 while Bellman-Ford-Moore and Johnson used 840 and Floyd-Warshall used 852 as shown in Figure 6. A comparative analysis of all the existing methods and our proposed method is depicted in the Table 3 and computational complexity is shown in Figure 7.

8.0 Conclusion

In this research work, the model of Nigeria road network from multiple sources to multiple destinations through several intermediary nodes (states) was adopted from the work of Ikpotokin and Tamber (2017) with its linear programming problem formulated which had 441 variables and 75 constraints. Due to the large number of variables and constraints, it was difficult to solve this manually. Hence we employed the use of LINGO computer application package which solves any LPP directly. We also adopted an algorithm for computing and analyzing networks adopted. The adopted algorithm confirms that, the shortest path obtained by the previous algorithms for a single source to single destination and single source to multiple destinations are valid for multiple sources to multiple destinations, but the previous algorithms takes longer time in solving similar problem MS-MD by splitting it into SS-SD before solving then after solving, the split networks are superimposed to form the MS-MD. The adopted algorithm is straight forward and unambiguous. In general, when applied to other road networks of multiple sources to multiple destinations, less number of iterations and floating point arithmetic are used to obtain the shortest route from the various sources to various destinations without splitting. Further, the adopted algorithm handled the other types of networks like single sources to single destinations, single sources to multiple destinations and multiple sources to single destinations directly, without splitting as well as critical path problem of multiple projects without altering the order of the network like the case of the existing algorithms.

This study did not consider any network with negative weights, negative circles, and we did not attach any penalty to the network.

We recommend that, researchers, intending researchers, decision makers, transporting companies, transporter, telecommunication companies, computer networkers, petroleum processing and distributions companies, GPS (Global Positioning System) service providers, electricity distributors and marketers, construction companies, network marketers, bioinformatics, extension workers, security agencies and large-scale farmers and others should adopt the adopted algorithm which saves resources in order to optimize resources.

9.0 Research Findings

In this research work we found that:

- The network model was most suited for the dynamic programming algorithm because it shows the link between all the states (nodes), directed network and has only positive edge weights.
- Linear programming problem of the adopted model was formulated with 441 variables and 75 constraints. Due to the large number of variables and constraints, it was difficult to solve this manually. Hence we employed the use of LINGO computer application package which solves any LPP directly.
- The dynamic programming algorithm was the most suited method for solving complex problems because, it break-down complex problems into individual sub-problems and solve them.
- The existing algorithms were used to obtain the shortest path problem of the model designed and developed (but only for the SS-SD, SS-MD and MS-SD networks).
- The adopted algorithm enhanced the speed and accuracy of solving network problems of multiple sources-multiple destination.

- Compare to the existing algorithms, the adopted algorithm has the ratio of 1: N_s: N_s x S_n of our adopted method: single – multiple; single – single for solving multiple –multiple problems respectively.
- The adopted algorithm also handled the other types of directed graph (with non-negative edge weights) of: single source – single destination, single source – multiple destinations and multiple sources – single destination.
- The adopted algorithm is very stable and be implemented on network analysis of shortest path problems and critical path analysis.
- The adopted algorithm can also be used to obtain the critical path (i.e. it can be used to solve the project management problem) of single source – single destination, single source – multiple destinations, multiple sources – single destination and multiple sources – multiple destinations.
- Dynamic programming algorithm was modified in the steps.

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COVID-19: Imperative for E-Service Quality Improvement in Business Organizations

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Abstract
The emergence of COVID-19 pandemic has brought about several challenges to business organizations globally. One of the major concerns of COVID-19 is the delivery of e-services. Due to the lockdown measures to reduce the spread of the corona virus, most business organizations resorted to e-channels to render services to customers. This has implications for e-service quality management and the way customers perceive and evaluate the quality of services. To deliver high e-service quality, there is need for business organizations to understand the dimensions of e-service quality and how to improve on the dimensions on a continuous basis. Extant literature reveals that several dimensions have been proposed to measure the perception of e-service quality. The paper reviewed these dimensions and extracted the key e-service quality dimensions that business organizations can focus on to obtain feedback on their performance and to improve the quality of e-services. Based on the review, the paper proposed seven e-service quality dimensions: website appearance, ease of use, reliability, fulfillment, responsiveness, personalization and security. It is expected that these e-service quality dimensions will provide a frame of reference for businesses managers to assess their service performance and to identify the aspects of e-service delivery that need improvement.

Keywords: COVID-19, e-service quality, perception, performance, e-channels

Introduction

The worldwide COVID-19 pandemic has led to alterations in the operation of businesses (KPMG, 2020). It has changed the service delivery landscape and the way business organizations render services to their customers across the world. During the lockdown period to combat the spread of the virus, most businesses moved their services from the four walls of the business to online platforms. With restrictions in the movement of people, there was disruption in normal operating conditions. Employees were asked to work from home and customers were buying and receiving services online to reduce face-to-face interactions in service delivery. Thus, COVID-19 emphasizes the need for businesses to be more technologically driven. Technology has a remarkable influence on the growth of self-service and service delivery options in business organizations (Dabholkar & Bagozzi, 2002). While some sectors were prepared for online service delivery some were not prepared. However, the COVID-19 experience confirms the assertion of Yousif (2015) that all businesses compete in two worlds, the physical or tangible world that can be seen and the electronic world that is made up of information through the internet or other business networks. This fundamental change has raised several service quality issues and so interest in managing services from customers' perspective is considerably high. Therefore, there is need to understand specifically customers' perceptions of e-service quality and how businesses can improve on e-service quality delivery.

It has been noted that the key strategy for success and survival of any business in a competitive situation is to deliver quality services to customers (Fariz & Bagher, 2014). Customer perceived e-service quality is one of the critical determinants of the success of online business (Yang, Jun & Peterson, 2004). Consequently, service quality has become a major area of attention during the past few decades for managers, researchers and practitioners because of its huge impact on business performance (Biljana & Jusuf, 2011). E-services provide a unique opportunity for businesses to offer new models for service design strategies and new service development. On the other hand, it provides customers with a superior experience with respect to the interactive flow of information. As noted by Iham, Ahmad, Low and Hamid (2013) e-service quality is one of the significant factors that play a major role in the success or failure of business organizations. It has become a key strategic factor for businesses to differentiate their services from other competitors. When businesses provide high service quality, it increases efficiency and effectiveness in service delivery which in turn lead to increase in profitability of the business. In addition, the provision of better services might result in repeated purchases and extended positive word of mouth.

Poon & Lee (2012) are of the view that the fundamental reason for using e-service instead of traditional service is the convenience of being able to receive the service at one’s comfort and being able to access the service whenever one wishes. Mohammad, Rushami, Rabiul and Abdullah (2013) opined that to deliver quality services electronically, business organizations need to understand the perception of customers regarding the quality of their services and the way customers evaluate them. Mohammad, Mohsen and Roza (2013) also noted that to ensure high perceived service quality by customers in electronic markets, it is necessary for businesses to evaluate the level of service quality being offered. According to Zeithaml, Parasuraman and Malhotra (2000) managers need answers to many questions such as what is a good service on the web? What are the underlying dimensions of superior e-service quality? How can e-service quality be conceptualized, measured and thereby assessed? What actions can be taken to deliver e-service quality? And, what role will different technologies play in addressing the various aspects of customer service on the web? The issue of service quality is being recognized as strategically important for managers of businesses with a web presence, as more and more customers are taking the plunge and engaging with businesses over the Internet (Collier & Bienstock, 2006).

The aim of this paper is to emphasize the need for e-service quality improvements in business organization as a response to COVID-19 pandemic. The focus is to identify and discuss the key service quality dimensions for e-channels that business organizations can focus on to improve the quality of e-services. If businesses have knowledge about the quality attributes they can use to measure the quality of their e-services and how it affects its use by customers, it would be much easier for them to take necessary measures and steps to improve the overall service quality. The paper is organized as follows. First, it describes the concept of service quality. Then, it discusses the concept of e-service quality. Thereafter, it examined the measurement of e-service quality by researchers and then it identified key e-service quality dimensions that businesses can focus on in improving e-service quality. It concludes by emphasizing the need to incorporate and improve on the identified dimensions for increased business performance.

The Concept of Service Quality

Service quality describes a service that fulfills the expectations of customers and satisfies their needs. Several authors (Biljana and Jusuf, 2011; Zehir and Narcıkara, 2016; Al-hawary and Al-Smeran, 2017) have noted the importance of service quality to a business and have demonstrated its relationship with profits, increased markets share, return on investment, customer satisfaction and future purchasing intention. It offers competitive advantage through customer attraction and customer retention. Thus, it is an important concept in business organizations and services marketing. Service quality has been investigated and explained from two perspectives: traditional service quality and e-service quality. Unlike the quality of a good, service quality is an abstract construct that is difficult to explain and measure (Lee & Lin, 2005). At the traditional level, the most extensively used dimension of service quality is the one developed by Parasuraman, Zeithaml and Berry (1988). Initially, Parasuraman, Zeithaml and Berry (1985) identified ten dimensions of service quality which include reliability, responsiveness, competence, access, courtesy, communication, credibility, security, understanding/knowing the customer and tangibles. However, Parasuraman *et al.* (1988) conducted empirical studies in several industries to develop and refine the dimensions of service quality and to quantify customers' global as opposed to transaction-specific assessment of a company's service quality. Based on scale enhancement, the initial ten dimensions were reduced to five dimensions. These dimensions popularly referred to as SERVQUAL were conceptualized as follows:

Tangibles: The appearance of physical facilities where the service is provided, including the design of the office, equipment for providing the service and appearance of personnel.
Reliability: The ability to perform the promised service dependably and accurately, consistency in performance and dependability of the service. This means that the business performs the service right at the first time and it honors its promises.
Responsiveness: The willingness of employees to help customers and provide prompt service.
Assurance: The knowledge and courtesy of employees and their ability to inspire trust and confidence.
Empathy: The caring and individualized attention the business provides its customers.

The five dimensions of SERVQUAL have been used to determine service quality in various service industries and were found to be valid in the traditional service environment however, they have been found unsuitable to determine the quality of services delivered over the web and other technology-based services. Despite the wide application of SERVQUAL model for determining service quality, some researchers (Hongxiu, Yong and Reima, 2009; Barrutia & Gilsanz, 2009; Iham *et al.*, 2013) have raised some concerns about the conceptualization and operationalization issues of the model for determining e-service quality. This is because the service delivery processes of e-services differ significantly from that of traditional service. E-service is different from traditional service based on two major attributes of e-service which are the absence of staff and self-service by customers. Due to the distinctive characteristics of e-services, measuring e-service quality differs from measuring traditional service quality (Hongxiu *et al.*, 2009; Ghorbani & Yarimoglu, 2014). Therefore, the attributes for defining a high quality service are expected to differ in the two contexts. This was noted by Yang and Fang (2004) when they stated that the service quality dimensions that are relevant for traditional services may not be relevant for online services. This led to the search for the dimensions that determine the quality of e-services.

The Concept of E-Service Quality

The concept of e-service emerged from the growth of the internet and its application in business. From the year 2000, e-service practices have been on the increase and e-service quality models have been evolving (Yarimoglu, 2014). This development brought about the issue of e-service quality and several authors have offered a variety of definitions. Zeithaml *et al.* (2000) defined e-service quality as the extent to which a website facilitates efficient and effective shopping, purchasing and delivering of products and services. In the view of Parasuraman, Zeithaml & Malhotra (2005) e-service quality encompasses all phases of a customer's interactions with a website including the extent to which a website facilitates efficient and effective shopping, purchasing and delivery. As noted by Ojasalo (2010) an e-service operation is one where all or part of the interaction between the service provider and the customer is conducted through the Internet. E-service has also been described as the total evaluations and ideas of consumers considering the e-service delivery privilege in the virtual marketplace (Iham *et al.*, 2013). It has also been defined as a service delivered to a customer or a potential buyer through a website (Sakhaei, Afshari & Esmaili, 2014). Generally, e-service is the electronic provision of services by businesses to customers.

Several researchers (Zeithaml *et al*., 2000; Santos, 2003; Parasuraman *et al*., 2005; Hongxiu *et al*., 2009) have focused on conceptualizing and measuring e-service quality and examining its effects in the electronic market space. It has been noted that customers' perception of the quality of a service depends on customers' pre-service expectations (Parasuraman *et al*. 1988; Zeithaml *et al*., 2000). Customer expectations are partial beliefs or assumptions about a service that serve as standards or reference points against which service performance is judged. It is the evaluation of the service performance customers received according to whether it meets certain standards. The evaluation emanates from customers' comparison between prior expectations about the service and their perceptions after actual experience of service performance (Shirshendu & Sanjit, 2011). When customers are doing such evaluations, they may refer to their feelings as well as their cognition in the service consumption process. Thus, perceived service quality to a large extent, is what the customer says it is. This is because for a business to know whether it is providing service which is judged 100 percent satisfactory by 100 percent of its customers, it cannot make the judgement by itself. The business need to ask, observe and find out from the customers themselves. It has been noted that to increase e-service usage, existing quality standards need to be adapted to provide means to assess, assure and improve the quality (Batagan, Pocovnicu & Capisizu, 2009).

Measurement of E-Service Quality

Service quality is usually measured with a set of dimensions. Service quality dimensions are a set of features that describe customers' experience with a service. Recently, the measurement of service quality of e-channels has become a major issue facing business organizations. Therefore, research into how customers perceive and evaluate the quality of e-services is still ongoing. Several researchers have tried to identify the dimensions of e-service quality. These dimensions try to define e-service quality with the help of specific website characteristics of a given service. According to Zeithaml *et al.* (2000) and (Raman, Stephenaus, Alam & Kuppusamy, 2008) additional dimensions to SERVQUAL are needed in order for the full construct of e-service quality to be captured. Consequently, the measurement of e-service quality has been receiving increasing attention. According to content review of the literature in this area, several studies have tried to highlight the dimensions of e-service quality linked with different types of e-service such as e-banking, e-shopping, e-retailing, e-ticketing, e-booking, e-brokerage. Zeithaml *et al*. (2000) identified ten dimensions of e-service quality: reliability, responsiveness, access/flexibility, ease of navigation, efficiency, assurance/trust, security/privacy, price knowledge, site aesthetics and customization/personalization. In the view of Liu and Arnett (2000) the dimensions of e-service quality include system use, system design quality, information quality and playfulness. Kaynama and Black (2000) developed an E-QUAL scale to measure the service quality of online travel service, and found that the most important seven dimensions of e-service quality are content and purpose, accessibility, navigation, design and presentation, responsiveness, background, personalization and customization.

Cox and Dale (2001) identified six dimensions of e-retailing which include website appearance, communication, accessibility, credibility, understanding and availability. Loiacono, Watson and Goodhue (2002) develop the WEBQUAL scale which comprised twelve dimensions including information adaptability, trust, design, visual requirement, flow, business process, interaction, response time, intuition, creativity, overall communication and replaceability. Zeithaml, Parasuraman & Malhotra (2002) revised the earlier ten dimensions of Zeithaml *et al*. (2000) and reduced them to seven. They include efficiency, reliability, fulfillment, privacy, responsiveness, recovery and contact. Madu and Madu (2002) proposed fifteen dimensions of e-service quality as performance, features, structure, aesthetics, reliability, storage capacity, serviceability, security and system integrity, trust, responsiveness, service differentiation and customization, web store policies, reputation, assurance and empathy. Barnes and Vidgen (2002) identified five dimensions which include usability, design, information, trust and empathy. Wolfinbarger and Gilly (2002) developed an e-service quality scale which was initially titled COMQ and later was renamed eTailQ in 2003 with the following four dimensions: website design, fulfillment/reliability, security/privacy and customer service.

Santos (2003) identified ten dimensions of e-service quality which include ease of use, appearance, linkage, structure and layout, content, reliability, efficiency, support, communication, security and incentive. Yang and Fang (2004) identified four key dimensions of e-service quality of online securities brokerage services which include responsiveness, reliability, ease of use and competence while Yang *et al.* (2004) identified six e-service quality dimensions of reliability, access, ease of use, attentiveness, security and creditability. Parasuraman *et al.* (2005) proposed the E-S-QUAL scale (core dimensions) with four dimensions: efficiency, fulfillment, system availability and privacy and the E-RecS-QUAL scale (recovery dimension) with three dimensions: responsiveness, compensation and contact. This new dimension indicates that recovery is an important aspect of service quality assessment by customers. Lee and Lin (2005) developed instrument dimensions of e-service quality by modifying the SERVQUAL model in the online shopping context. Five dimensions of e-service quality identified include website design, reliability, responsiveness, trust and personalization.

Moreover, Raman *et al.* (2008) identified six e-service quality dimensions which include ease of use, appearance, reliability, customization, communication and incentive. Swaid and Wigand (2009) are of the view that the dimensions of e-service quality are website usability, information quality, reliability, responsiveness, assurance and personalization. Hongxiu *et al.* (2009) identified nine dimensions of online travel service which include ease of use, website design, reliability, system availability, privacy, responsiveness, empathy, experience and trust. Ojasalo (2010) proposed eight dimensions which are ease of use, web site design and appearance, personalization, information, responsiveness, communication, security, and reliability. Poon and Lee (2012) identified seven dimensions including efficiency, trust, convenience, personalization, layout and appearance, content and accessibility. Farnaz, Mohd, Ahmad, Norhayati, Ahamad & Mohsen (2012) investigated internet banking service quality and identified efficient and reliable services, fulfillment, security/trust, site aesthetics, responsiveness/contact and ease of use. Fariz and Bagher (2014) evaluated and ranked the factors influencing the quality of e-banking service based on both active and passive dimensions. The active dimensions are website content, website composition and structure, website links, website ease of use and website appearance while the passive dimensions are motivation, website reliability, website performance, website support, website communication and website security. Yousif (2015) identified information quality, software quality, website quality, electronic ethics and service diversification quality while Al-Hawary and Al-Smeran, (2017) identified six dimensions: reliability, ease of use, efficiency, website design, privacy and responsiveness

From the review, it can be observed that over the years, researchers have developed several dimensions of e-service quality with the objective of identifying those aspects that are more important for e-services. It can be concluded that there is no consensus among researchers with regard to the important dimensions of e-service quality. So, there is need to develop a concise list of dimensions that can be applied for measuring e-service quality irrespective of the type of e-service.

Key E-Service Quality Dimensions

A number of researches have sought to identify the attributes of a service that contribute most to quality evaluations. However, the studies did not agree on a single set of dimensions. Several dimensions of e-service quality have been propounded to offer managers insights into the dimensions of service quality that can be used to improve service offering. Therefore, identifying key e-service quality dimensions will provide a frame of reference for businesses to assess their service performance. Drawing on the above literature analysis and the peculiarity of e-services, this paper proposes seven e-service quality dimensions assumed to be highly relevant in the context of e-services. They are website appearance, ease of use, reliability, fulfillment, responsiveness, personalization and security. Although some researchers (Iham *et al.*, 2013; Yarimoglu, 2014) are of the opinion that the measurement of e-service quality should be industry specific, there should be some key e-service quality dimensions that business organizations can focus on irrespective of the type of e-service. The key e-service quality dimensions identified are discussed as follows:

Website appearance

Website appearance is the visual appearance and audible applications of a site. It refers to how the site looks like and it includes the site aesthetics, information structure, colour, animation, pictures, text, format, sound and visual design. The website is the first point of contact a customer is faced with when accessing online services. It should have a clear structure that enables users to find important information at first sight, text has to be displayed legibly with proper font size and print size, images or symbols should be readily identifiable (Poon & Lee, 2012). Website appearance has influence on customer perceived image of a business and can attract customers to purchase online (Hongxiu *et al.*, 2009). As noted by Fariz and Bagher (2014) all items should be explained in simple and clear language on the website so that it is understandable to most users. It should avoid website links or references to non-related contents and it should be clear and well organized. According to Lee & Lin (2005) the user interface should be visually appealing and tidy to make it attractive to customers. Wolfinbarger and Gilly (2001) opined that availability of information is one of the most important features of online purchasing. The authors further noted that if we compare online and offline purchasing, online purchasers have an edge due to the fact that they receive information directly from the website without having to seek the help of a salesperson. Therefore, adequate and accurate information that do not require high expertise on the part of customers to understand is a major part of the service delivery process.

Based on previous studies, website appearance plays a significant role in how customers judge service quality (Cox & Dale, 2001; Santos, 2003; Lee & Lin, 2005). Rosen and Purinton (2004) demonstrated how website appearance affected not only online customers' impression of the website but also intentions for revisit. Website is the starting point for customers to gain access to the business organization and it can influence customers' perceived image of a business and attract customers to engage in online services. As such, the website interface should be well designed and visually appealing. Raman, *et al.*, *2008*) opined that a website with a lot of flash animations, pop-up advertisement and graphic banners would dissatisfy the user. They noted that online customers are only interested to engage in whatever they want to do rather than seeing animations.

Ease of Use

Ease of use is the degree to which the e-channel can be understood and operated in a simple and easy way. It can also be described as how easy it is for customers to access the site. Ease of use also refers to the ability of a customer to find information or enact a transaction with the least amount of effort. It is measured by the ability of customers to get to the website, find their most wanted product/information and conveniently logout. It includes aspects such as functionality, accessibility of information, ease of ordering and navigation. Ease of use has been found to be one of the major determinants of e-service quality (Shirshendu & Sanjit, 2011; Narteh, 2013). Yazeed, Yazidu and Ibrahim (2014) noted that some users find the instructions on how to perform some operations quite difficult to understand and so, if users feel that a system is easy to use, then the chances of using the system will be greater. Ease of use also includes customers' perception of the degree of user friendliness of the online channel and the ease with which a user is able to interact with an information system. A user-friendly site has features of quick response time, easy navigation, rich information content and responsive interaction with customers (Al-Hawari, Hartley and Ward, 2006). A user-friendly website plays an important role in increasing web users' satisfaction which increases the probability of obtaining loyal customers. On the other hand, when the website is not easy to use, it may prevent users from finalizing the desired transaction and consequently, the users may not revisit the website.

The issue of navigation is critical for e-services. Navigation is having consistent menus that lead to key pages on a site. A clear navigation aid allows visitors to know where they are on a site and provides the ability to find their way back to a previous menu screen. Ease of use also includes effective search engines, the ability to easily change or cancel an order, and the ability to inform customers of missing information. Such sites should make it easy for customers to perform all the functions without asking for help. If a customer finds a site to be difficult to use, then the dissatisfaction of the experience will affect future behaviour to re-patronize the online channel (Collier and Bienstock, 2006). As noted by Raman *et al.* (2008) e-service providers should focus on website navigation and applicable functions to cater for the needs of various user segments. For the first time user, the e-service provider should provide simple site that will not lead into further confusion. Most first time users do not know where to start from. To ease the difficultness and to reduce the confusion, the e-service should provide convenient attributes or features to deliver the instructions.

Reliability

Reliability relates to accuracy, speed and constant availability of a service (Muhammad, Aslam, Afgan, & Abbasi, 2014). It is the ability of the online platform to perform the promised service dependably, consistently and accurately (Narteh, 2015). Reliability is concerned with the technical functioning of the site, particularly the extent to which it is available and functions properly, ability to perform the promised service dependably and accurately, the degree of accuracy and consistency of performance, performing the services at designated time and the provision of accurate information (Zeithaml *et al.*, 2002). It also means that the business honours its promises. Specifically, it involves: accuracy in billing, keeping records correctly, performing the service whenever there is request. Reliability means that the site should function all the time and it should be available 24/7 as promised. In e-service quality research, reliability has been found to be the most significant determinant of e-service quality (Narteh, 2013; Al-Hawary and Al-Smeran, 2017) and customer satisfaction (Mwatsika, 2016). The importance of reliability is based on the premise that customers' perception of e-service quality is likely to increase when the service is performed as promised or expected by the customer. If customers cannot use the online system when they need online service, they may deviate from using it. Reliability is vital to make customers to trust that the business will perform what it has promised to deliver (Wali & Opara, 2012).

Fulfillment

Fulfillment is the extent to which an e-channel performs outcomes which meet customer's expectation in terms of the extent to which the site's promises about order delivery and item availability are fulfilled (Ojasalo, 2010). It represents the outcome performance of service delivery and the focus is on customers' requirements in terms of the purpose for using the e-channel and what they actually receive. Therefore, the e-channel should deliver what it promises to deliver. Fulfillment also involves accurate display and description of a product so that what customers receive should be what they ordered for as well as the delivery of the right product within the time frame promised. Some authors (Wolfinbarger and Gilly, 2003; Farnaz *et al.* 2012) have found fulfillment to be a determinant of e-service quality.

Responsiveness

Responsiveness means effective handling of problems and returns through the site. It has to do with how customer care or support service responds to help customers when they face problems with a service (Muhammad *et al.*, 2014). It is the business ability to handle customer complaints due to transactional failures. It includes the extent to which the business have put in place measures to recover services when the e-channel could not deliver as expected and the ability of the business to handle customer complaints arising as a result of transactional problems. Customers of e-services expect quick feedback on requests and so it also involves the attention and promptness in dealing with customer requests, questions, complaints and problems as well as compensating customers when they encounter financial losses. It should be noted that e-service quality is more than just how a customer interacts with a website, it also relates to customers' perceptions of the outcome of the service along with recovery perceptions if a problem should occur (Zeithaml *et al.,* 2002; Parasuraman *et al.,* 2005). Therefore, the ability to handle questions, concerns and frustrations from the customer is essential to the perception of e-service quality. Madu and Madu (2002); Al-Hawary and Al-Smeran (2017) found responsiveness to be a critical factor in e-service quality.

Personalization

Personalization refers to customer perception of the individualized attention and differentiated service that are tailored to meet individual's needs and preferences (Lee & Lin, 2005). It is the ability of the online channel to address users on a one-on-one basis. Personalization involves an understanding of customer needs and preferences so that the content of the web pages give the feel that it is specifically designed for them by acknowledging repeat customers by their names whenever they login to the website. This provides customers with the feeling of familiarity and closeness, thus positively influencing customer relationship and customer loyalty (Kaynama & Black, 2000; Poon & Lee, 2012). One benefit of e-channel is that the website can be personalized to the user's needs. The lack of real-time interaction tends to prevent potential customers from purchasing and receiving a service through online platform (Lee & Lin, 2005). Personalization may be done based on past purchases as well as other information provided by customers (Ojasalo, 2010). E-service enables a business to collect and store information about its customers and identify them at individual level. If customer database is linked to the website, then whenever they visit the site, the business can greet them with targeted offers. The more they buy or receive the service online, the more the business can refine their profile and market to them effectively. Personalization should be based on the user's needs so as to create an enjoyable interface for each user.

Security

Security is the protection of customers from fraud and financial loss as well as the protection of customers' personal information (Narteh, 2015). Although security concerns differ among countries, e-channels can be accessed globally and so, Lee & Lin (2005) opined that online channels should provide secure online transactions to make customers feel comfortable when using it. This dimension holds an important position in e-service because customers perceive significant risks in the virtual market space due to the high prevalence of internet fraud (Zhengwei & Jinkun (2012). Customers' perception of risks tends to be high for online services especially financial services because customers believe that the internet payment channels are not secure and can actually be intercepted, which reduces the customers' level of trust. This tends to discourage them from engaging in online information search and making online banking transactions. It has been noted that customers will not embrace e-commerce until they have confidence that their use of services on open networks are secure and reliable; that their transactions will be safe and private; that they will be able to prove the origin, receipt and integrity of the information they receive; that they can identify those they are dealing with; and that there are appropriate mechanisms available to them if something goes wrong. According to

Agbonifoh, *et al.* (2007) two kinds of security are desired by customers who use the internet, namely, informational and transactional securities. Informational security is associated with safety from any kind of loss arising from illegal use of customers' information by unauthorized persons while transactional security refers to safety over business deals carried out over the internet. Security has been found to be an important dimension of e-service quality (Madu & Madu, 2002; Wolfinbarger & Gilly, 2003; Akinmayowa & Ogbeide, 2014).

E-Service Quality Improvements

The virtual market space allows customers to easily compare the benefits of competing services and to easily switch due to its low switching costs. One of the major factors that will make customers to continue using e-channels is service quality. Businesses should therefore, try as much as possible to make customers satisfied with their services and offerings online because delivering effective e-service quality is essential for customers to use the service and this can be achieved by measuring and improving the dimensions of e-service quality. This is necessary because a high level of e-service quality contributes to achieving business goals in terms of better and more efficient relationship with customers, increased ability to attract potential customers, greater competitiveness and increased benefits in the long-run (Barrutia & Gilsanz, 2009). In the view of Aaker (2011) a high level of perceived service quality has impact on the perception or superiority of a service over its alternatives. E-service quality not only encourages customer loyalty but also may change traditional buyers to online ones, thereby reducing operating costs. The regularity and consistency of using online channels depends on the pleasure and effectiveness of customers' perceptions about electronic experience. Therefore, e-service providers have to practice on-going measurement of customers' preferences and expectations and constantly monitor and improve e-service quality.

In an interpersonal service encounter, where customers have direct contact with service personnel, the way service personnel behaves, talks, smiles and attend to customers, will influence to a high extent the satisfaction of the customers with the service delivered. In the virtual space, customers communicate with the business through an information system. By using the internet as a service delivery channel, businesses should be aware of the fact that some aspects of the human interaction of traditional service settings cannot be replaced by technology. Such aspects, according to Cox and Dale (2001) are for example courtesy, friendliness, helpfulness, care, commitment, flexibility and cleanliness. The absence of these aspects of human interaction through which quality can be delivered to customers will have to be compensated for by other quality factors. The fact that e-services are delivered over the Internet pose some challenges to the service providers. First of all, the direct contact between service employees and customers is missing and secondly the service delivery setting is completely changed. In the case of e-services, websites become the "moment of truth" between customers and the business. Due to the lack of face-to-face interaction with service providers, the user interface is what customers of e-services interact with and as such it can be expected to influence their evaluation of the overall service quality. That is why it is advisable that businesses should consider and improve on the design and function of their websites. It should be designed for customers' ease of use, including searching, navigating and use (Hongxiu *et al.*, 2009). Customers may get frustrated and eventually be discouraged from visiting the website if it cannot be accessed easily or it is very slow. The graphics, colours and images should be attractive enough to the customers. Additionally, the information content of the website should be updated regularly to provide up-to-date information for the customers.

Apart from the website, there is need to constantly monitor the performance of the site in terms of reliability in service delivery, fulfillment and security. This will enable the business to quickly identify shortcomings and improve on them as quickly as possible. It should also be noted that technology can malfunction anytime. Therefore, businesses should be responsive when issues arise from using the online channel. If a customer had problems completing an online transaction and the business is not willing to render assistance, then, the customer may discontinue the use of online channels. Every business rendering services to customers online must have support e-mail or phone numbers that customers can call to receive help.

Conclusion

The measurement of e-service quality is a strategic issue in business organization that would like to survive in an increasingly changing business environment and competitive market space. The COVID-19 pandemic has made customers to be more sophisticated in the use of technology and is most likely that they will demand for increased higher standards of service delivery. Delivering quality service requires taking into account the importance of service quality dimensions and ensuring that customers are satisfied. Several e-service quality dimensions were highlighted and discussed based on the literature and there no agreement by researchers about the dimensions of e-service quality. A lot of e-service quality dimensions have been proved empirically to be important in online service delivery. Therefore, this paper attempted to extract the key dimensions of e-service quality and examined how these dimensions can be improved upon to enhance online service performance. Delivering high e-service quality requires an understanding of the e-service quality dimensions, and trying to improve the quality of the services, so that customers' continued use of the online channel is assured. The seven dimensions identified in this paper (website appearance, ease of use, reliability, fulfillment, responsiveness, personalization and security) will enable business organizations to understand how customers evaluate the quality of e-services. This will enable them to obtain feedback on their performance on e-service quality and also provide a tool for improving their e-service quality.

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Resilient Health Systems to Mitigate Impact of Covid-19 Pandemic on Medical Professionals

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Abstract

The COVID-19 pandemic marks an extraordinary global public health crisis with its rapid spread worldwide and associated mortality burden. The longevity of the crisis and disruption to normality is unknown. With COVID-19 set to be a chronic health crisis, Medical practitioners will be required to maintain a state of high alert for an extended period. The support received before and during an incident is likely to influence whether they experience psychological growth or injury. Though the entire medical workers have been the heroic face of the pandemic, the health systems and providers discovered that their workforce planning and deployment models had limitations. The trust of this study is to identify and evaluate models to overcome the challenges of physical, mental, and emotional exhaustion during and post covid-19 pandemic period. The outcome reveals most hits are medical practitioners who are directly involved in the diagnosis, treatment, and care of patients with COVID-19 and are at risk of developing psychological distress and other mental health symptoms. The increasing number of confirmed and suspected cases, overwhelming workload, lack of personal protection equipment, lack of specific drugs, and feelings of being inadequately supported may cause the mental problems of these medical practitioners.

The suggestive outcome is to build an Institutional and health system governance to digitally transform supply chain models and other middle and back-office functions to be more resilient and responsive to changing clinical priorities.

Keywords: Covid-19 pandemic, Medical workers, Resilience, Psychological distress

Introduction

The COVID-19 pandemic has placed unprecedented pressure on health systems. Containing and mitigating the spread and infection rate of the virus is essential, as is boosting the resilience of health systems to deal with both the ongoing outbreak and future crises.

Pandemic preparedness and response are quintessential “global public goods”: unless the new coronavirus is controlled everywhere, the risk of pandemic resurgence remains.

This implies the need for development of co-operation to shift from disease-focused actions to a systemic approach. The COVID-19 crisis is a threat to domestic health systems that are already short of resources in many developing countries. Firstly, the COVID-19 crisis could overwhelm health systems and ultimately challenge their capacity to handle basic functions. Secondly, it may lead countries to shift their resources away from tackling other diseases or services that cannot be delayed or reprogrammed such as maternal health services.

The following section reviews existing evidence and possible solutions in order for developing co-operation to build better resilience of health systems in response to the COVID-19 crisis.

Health system strengthening aims to ensure that the individual building blocks of the health system (i.e. service delivery, health workforce, information, medical products, vaccines, and technologies, financing, leadership and governance) function smoothly and interact well with one another. Universal health coverage, which allows everyone access to affordable and quality health services, is an important element of a strong health system.

Development of finance for health requires a shift in support towards more focused investments in transactional activities. It also needs to move from targeted support to specific communicable diseases, to more systemic resources to prevent chronic diseases (which are major risk factors and important prognostic factors in the face of the pandemic caused by covid 19) and also meeting the challenge of universal health coverage.

Effect of Pandemic on Healthcare Professionals

Coronavirus disease (COVID-19) pandemic has spread to 215 countries, with approximately **42,966,344 confirmed cases** of COVID-19 including **1,152,604 deaths**, reported by WHO as at October 27.

Frontline healthcare workers (HCWs) face a substantially higher risk of infection and death due to excessive COVID-19 exposure, resulting into physical and mental health impacts of COVID-19 pandemic on health-care workers (HCWs).

In Nigeria as at **Tuesday 17 Nov 2020, the NCDC data reads** 65,305 cases have been confirmed, 61,162 cases have been discharged and 1,163 deaths have been recorded in 36 states and the Federal Capital Territory, in sample tested of 717,942 (about 9%).

Health-care workers are crucial to any health-care system. During the ongoing COVID-19 pandemic, health-care workers are at a substantially increased risk of becoming infected with severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) and could come to considerable harm as a result. Depending on the phase of the pandemic, patients with COVID-19 might not be the main source of SARS-CoV-2 infection and health-care workers could be exposed to atypical patients, infected family members, contacts, and colleagues, or live in communities of active transmission.

The psychological responses of healthcare employees against this contagion are complex. Sources of distress may include emotions of vulnerability or loss of control, health of the family or others due to the spreading of the virus, changes of the working conditions, and environment or isolation anxieties. Healthcare professionals who are directly involved in the diagnosis, treatment, and care of patients with COVID-19 are at a risk of developing psychological distress and other mental health symptoms. The increasing number of confirmed and suspected cases, overwhelming workload, lack of personal protection equipment, lack of specific drugs, and feelings of being inadequately supported may cause the health problems of the healthcare providers.

RESILIENT HEALTH SYSTEMS SUPPORT

The existing Health systems are being overwhelmed by the COVID-19 outbreak. When health systems collapse, deaths from the outbreak itself can quickly be exceeded by deaths from preventable or treatable conditions, no longer managed by a failing health system. It is therefore crucial to maintain essential lifesaving health services even while addressing the pandemic. Health systems are being overwhelmed by demand for services generated by the COVID-19 outbreak. This collapse affects both direct mortality from the outbreak and avertable mortality from other conditions increase dramatically as well.

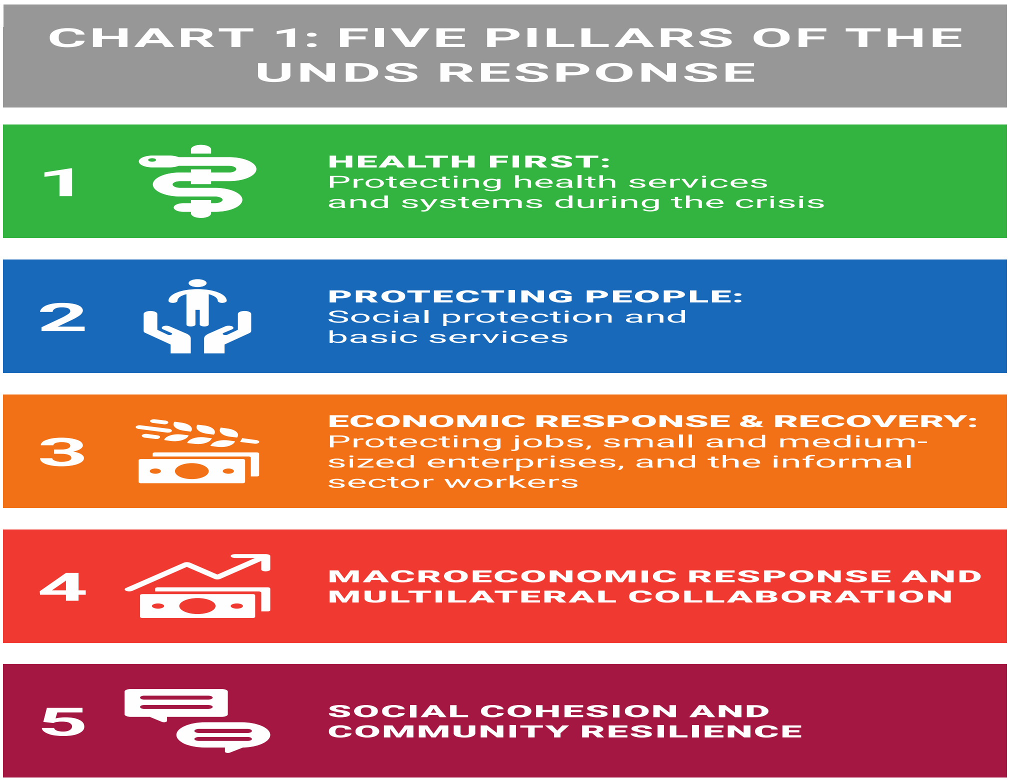
A Clear strategy to support and appropriately manage exposed and infected health-care workers should be in place, to ensure effective staff management and to engender trust in the workplace.

Policy makers need to strengthen health-care facilities in interpreting guidance during a pandemic that will probably be characterized by fluctuating local incidence of COVID-19 to mitigate the impact of this pandemic on their health officials.

Health system strengthening should not only focus on upstream support for governance and policy making but also downstream projects can be designed in a way that is conducive to strengthening.

Primary health care centers should be equipped with adequate health facilities, development of monitoring and evaluation systems, strengthen supply chain and ultimately a strong referral system to tertiary or reference hospitals to ensure proper management and improve patient, this will also help distribute the workload on the heath workers.

The United Nations Development System, whose activities makes up a significant portion of those of the United Nations also recommends Health Systems as the number 1 factor in their five pillars of support response as illustrated in the following diagram.



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Making available Personal protective equipment and other valuable medical equipment has become an important subject during the current coronavirus (COVID-19) epidemic. As COVID-19 is predominantly caused by contact or droplet transmission; Contact, droplet, and airborne transmission are each relevant during airway maneuvers in infected patients, particularly during tracheal intubation. Personal protective equipment is an important component of a system protecting staff and other patients from COVID-19 cross-infection. Appropriate use significantly reduces the risk of viral transmission.

This is as serious as a huge the lack of basic face mas for health professionals. For this reason, healthcare employees in Italy submitted posts through social media under the hashtag #GetMePPE requesting protective equipment such as masks, gowns, or face shields. In addition, nurses have also announced that they are planning to go on strike due to lack of protective equipment as well as the difficult of working conditions resulting from the spreading of the pandemic in the USA. And recently in Nigeria; Doctors in public hospitals went on strike over inadequate personal protective equipment alongside failure of payment of hazard allowance payment.

The effect of this inadequacies cannot be over emphasized as it will take a toll on the mental health of these health workers as they feel unprotected and not cared for while

fighting to keep the affected patients alive and ultimately, the risk of getting infected as well is remarkably high and this will reflect on the incidence of the pandemic .

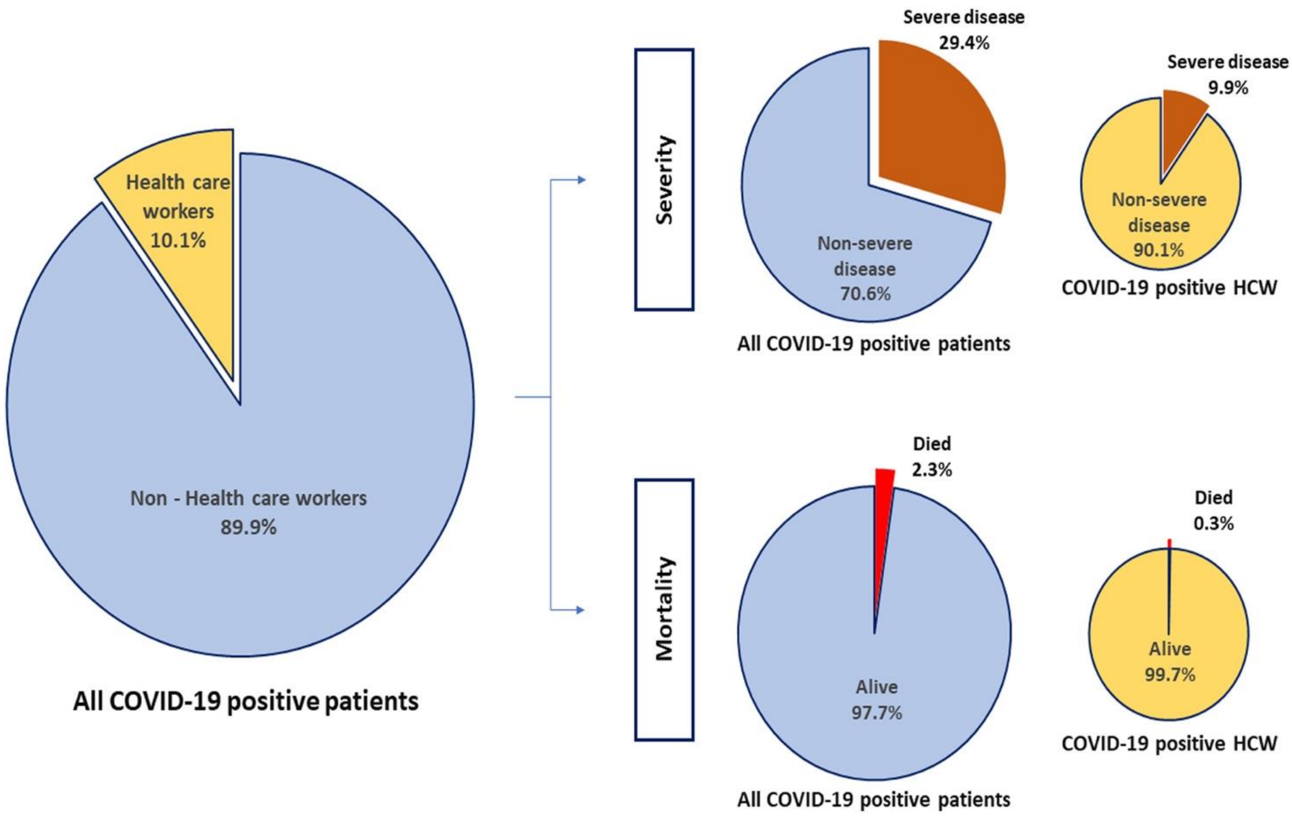
Quantitative data synthesis

In a research conducted and available in the **America journal of emergency medicine** shows combination of various test for analysis.

Highlights

- Meta-analysis of eleven studies showed that nearly 10% of COVID-19 positive patients are health care workers.
- The incidence of severe disease in health care workers (9.9%) was significantly lower than its incidence among all COVID-19 positive patients (29.4%).
- The mortality among health care workers (0.3%) was also significantly lower as compared to that of all patients (2.3%).

The total number of healthcare workers infected with COVID-19 (numerator) and the total number of COVID-19 positive patients (denominator) were extracted for calculation of proportion, and these proportions were summarised by using random-effects meta-analysis.



Outcome; Health care workers who are COVID-19 positive constituted a significant proportion of all COVID-19 patients; but the severity and mortality were lower among them.

In Nigeria; The National Association of Resident Doctors (NARD) has raised concerns about the number of infections and deaths among health workers.

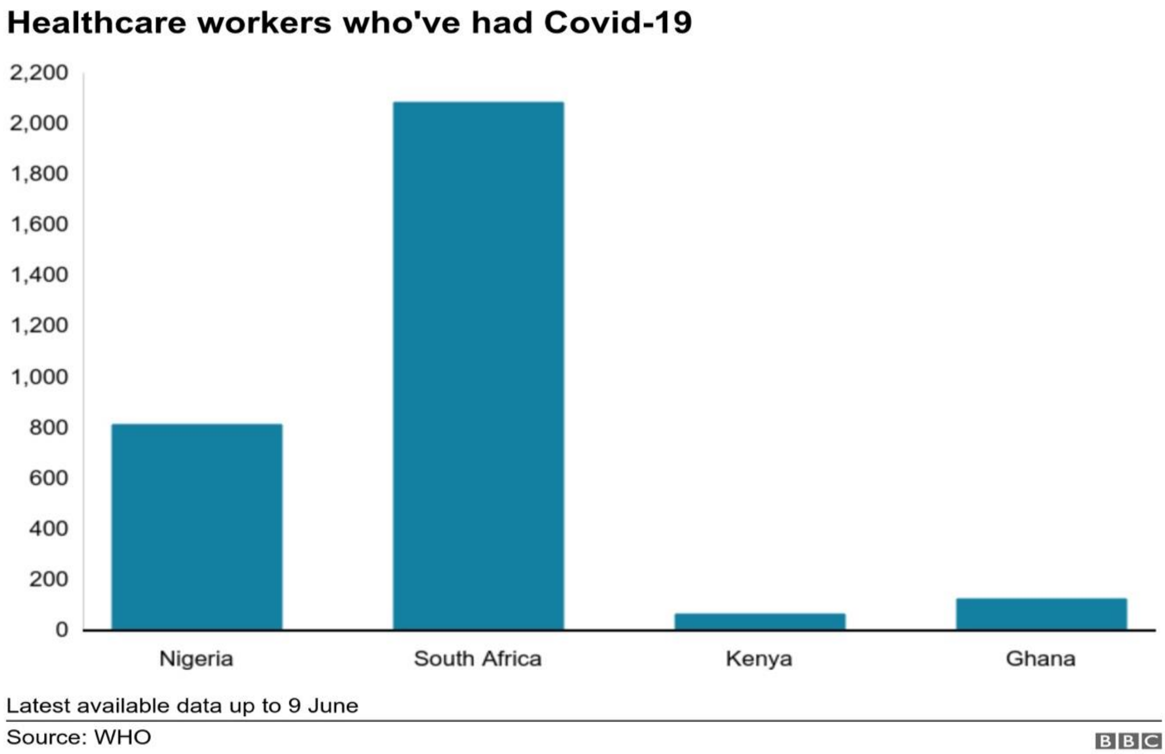
The Former NARD president Dr Olusegun Olaopa said: "We have lost quite a number of doctors to this pandemic."

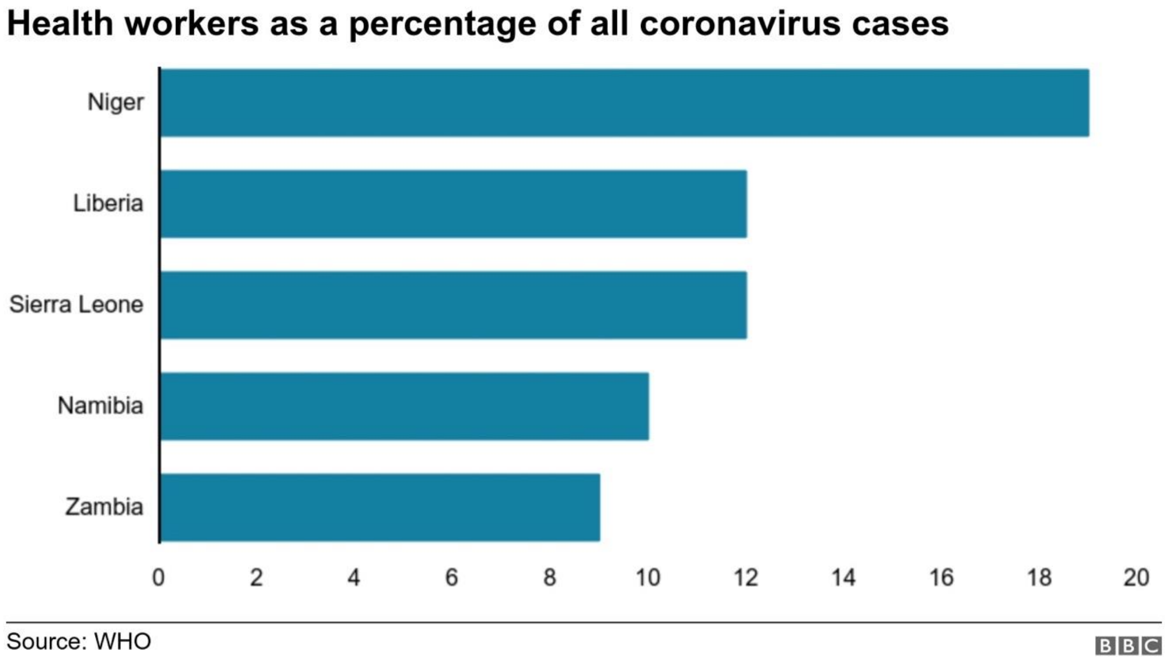
The World Health Organisation (WHO) ascertain this by saying health workers across the continent have been significantly affected by Covid-19.

Coming from studies provided by BBC reality check, More health workers have tested positive for coronavirus in Nigeria than in any other country in the continent, except South Africa. But they represent only 6% of all reported cases in the country, whereas in neighbouring Niger that figure is 19% - the highest in Africa.

Liberia and Sierra Leone have the next highest rates, with about 12% of total cases.

But these other West African nations all have much lower overall numbers of infections than their much larger regional neighbour, Nigeria.





Synergies especially exist between achieving universal health coverage and enhancing health security. For example, universal health coverage supports health security by preventing outbreaks through high immunisation coverage, providing early alert by facilitating access to healthcare for the entire population, better response from reliable infrastructure and healthcare workforce. At the same time, effective prevention and response to health crises is key to achieving universal health care. Health crises tend to divert resources to focus on crisis response and lowers trust in the health system. Therefore, they cause barriers to accessing healthcare. By avoiding and better managing such health crises, countries can ensure that patients have constant access to healthcare.

Recommendation

Resilient health systems are important for supporting response efforts during an infectious disease outbreak or natural hazard such as COVID-19, and help ensure the continued delivery of routine services needed by the community in non-emergency periods. However, health systems cannot achieve resilience without first identifying what capacities and capabilities are necessary to respond to the threats to the system.

World Health Organisation recommended preparation of health systems to mitigate the impact of the COVID-19 outbreak; The 16 recommendations reflect the characteristics of COVID-19, existing evidence and experience-informed practices in health system organization and financing, and emergent practices in the response to COVID-19 globally.

Table 1. Summary of 16 health system recommendations to respond to COVID-19

1. Expand capacity for communication and proactively manage media relations.	2. Bolster capacity of essential public health services to enable emergency response.	3. Clarify first-point-of-contact strategy for possible COVID-19 cases: phone, online, physical.	4. Protect other potential first contact health system entry points.
5. Designate hospitals to receive COVID-19 patients and prepare to mobilize surge acute and intensive care unit (ICU).	6. Organize and expand services close to home for COVID-19 response.	7. Maintain continuity of essential services while freeing up capacity for COVID-19 response.	8. Train, repurpose and mobilize the health workforce according to priority services.
9. Protect the physical health of frontline health workers.	10. Anticipate and address the mental health needs of the health workforce.	11. Review supply chains and stocks of essential medicines and health technologies.	12. Mobilize financial support and ease logistical and operational barriers.
13. Assess and mitigate potential financial barriers to accessing care.	14. Assess and mitigate potential physical access barriers for vulnerable groups of people.	15. Optimize social protection to mitigate the impact of public health measures on household financial security.	16. Ensure clarity in roles, relationships and coordination mechanisms in health system governance and across government.

Conclusion

The frontline healthcare workers are at risk of physical and mental consequences directly as a result of providing care to patients with COVID-19. Even though there are few intervention studies, early data suggest implementation strategies to reduce the chances of infections; shorter shift lengths, adequate protection in hospital environment, proper triage and quick diagnosis to reduce contact with suspected but undiagnosed cases. Mechanisms for mental health support and follow up of infected health care workers should be set in motion as it will cushion the long term effects of the pandemic on the mental health, even the entire wellbeing of Medical professionals. This will directly and indirectly improve the quality of health care service provided across board.

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Application of Queuing Model for Evaluation of Effectiveness of the Automatic Disinfectant Booths in Federal Polytechnic, Ilaro.

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ABSTRACT

The Federal Polytechnic Ilaro in preparation for resumption of academic activities and in compliance with government directive on preventive measure against the spread of covid-19 pandemic, installed automatic disinfectant booths at the school gate. The study examined the effectiveness of the disinfectant booth at the first gate of the Federal Polytechnic Ilaro. Direct observation survey research was engaged for collection of data with respect to arrival rate and service rate while multi channels queuing model was adopted to evaluate the effectiveness of the service with a view to give suggestion on possible improvement of the service. The study explored the interaction between arrival rate (λ) and services rate (μ) to determine the traffic intensity (utilization rate). It was revealed from the result of the study that the traffic intensity is less than 1, i.e. ($\frac{\lambda}{\mu c} < 1$) *which implies that the system was not explosive and that improvement would be easy. It was therefore suggested that technical personnel should be on ground at all time to attend to faults when the need arises, and constant power supply must be ensured for maximum utilization of the facilities.*

Keywords: *Covid-19 pandemic, preventive measure, traffic intensity, arrival rate, service rate.*

1. Introduction

It was like a nightmare between January and March 2020, and the whole world wished to wake up to be told it was just a dream, but rather faced with the dreadful reality of COVID-19 pandemic. The whole world was greeted with the full blown effect of the pandemic tagged COVID-19 which started in Wuhan city of China in 2019. The magnitude of the devastating effect seemed to have been undermined until it started spreading like wildfire through Europe, America and all other parts of the world. Eventually the grievous consequences of its presence was realized, necessitating drastic and decisive measures to be taken. In that vein it was crystal clear that movement should be restricted, business as well as public places such as schools had to be shut down to curtail the catastrophic effect that could have resulted from the spread.

Right from the moment of the lockdown and restriction, business began to count there loses in terms of profit, and academic institutions in terms of intellectual capacity decline and delay in human capacity development.

Though the number of causalities began to dwindle as preventive and curative measures were intensified but it was not until September that restriction and lockdown were relaxed. Though government approved the resumption of business and reopening of public places but not without condition and strict guidelines on preventive measures to curb the catastrophic effect of the spread of the pandemic.

The Federal Polytechnic, Ilaro, in compliance with government regulations on resumption of academic activities put up some measures which includes; [1] rescheduling of lecture periods from five to three days for all categories of students (the ND I and HND I for Monday, Wednesday and Friday while the ND II and HND II stated for Tuesday, Thursday and Saturday). This measure was put in place to have a controlled and manageable population since student population constituted over 90% of the human traffic in the institution [2] the lecture contact hours was reduced to one instead of two or three hours (that was with the understanding that the student had been engaged on e-learning during the restrictions and lockdown) [3] improved sanitation such as washing of hands and use of sanitizers in different offices within the institution [4] re-arranging of sitting in classrooms to conform with social distancing rule.

In addition to the above the first gate of the institution was restricted to pedestrian who were expected to be sanitized through an automatic disinfectant booths before access is granted into the institution. While the motorist were required to access through the second gate where they will be tested with infrared thermometer and compel to wash their hands before access is granted. The focus of this study is to evaluate the effectiveness of the automatic disinfecting booths with a view to give necessary recommendations to the management of Federal Polytechnic, Ilaro on possible ways of improvement.

2. COVID-19

The concept of the Virus Corona originated from Latin, it means crown. The infamous Coronavirus got its name from a spiky crown of glycoproteins on its surface. The World Health Organisation (WHO) explained Corona viruses as a large family of viruses that cause illness ranging from the common cold to more severe diseases such as Middle East Respiratory Syndrome (MERS-CoV) and Severe Acute Respiratory Syndrome (SARS-CoV). The most recent Coronavirus disease which was officially tagged by the Chief Scientist at WHO in Switzerland, Soumya Swaminathan, as COVID-19 to mean coronavirus disease of the year 2019, is a new strain that was discovered in 2019 and has not been previously identified in humans. The virus whose genome consists of a single strand of ribonucleic acid is zoonotic that transmitted between animals and people. The virus originated from a Chinese city, Wuhan, in December 2019 as reported by WHO.Most of the literature on coronavirus was documented and published in science and medical journals. Van der Hoek, Pyrc, Jebbink, Vermeulen-Oost, Berkhout, Wolthers, & Berkhout, (2004) human coronavirus, HCoV-NL63, sometimes in the year 2004 in addition to those in existence which includes human coronavirus 229E (HCoV229E), HCoV-OC43 and severe acute respiratory syndrome (SARS)-associated coronavirus (SARS-CoV). Similarly, in the year 2012, Zaki, Boheemen, Bestebroer, Osterhaus, and Fouchier (2012) reported the isolation of a novel coronavirus from a man with pneumonia in Saudi Arabia.

Coronaviruses have the potential to cause severe transmissible human disease, as demonstrated by the severe acute respiratory syndrome (SARS) outbreak of 2003. Bermingham et al. 2012 described the clinical and virological features of anovel coronavirus infection causing severe respiratory illness. Due to the zoonotic nature of coronavirus, it transmits easily with people, it keeps spreading among human and animals, Abroug et al. (2013) reported that in 2013 in every 3 persons in a family were found to be infected with MERS-CoV in Tunisia.In December 2019, Zhu et al. (2020) reported in research (Funded by the National Key Research and Development Program of China and the National Major Project for Control and Prevention of Infectious Disease in China), that a cluster of patients with pneumonia of unknown cause was linked to a seafood wholesale market in Wuhan, China which was later confirmed to be COVID-19. Different from both SARS-CoV and MERS-CoV, COVID-19 is the seventh member of the family of coronaviruses that infect humans. Presently, it has been established that human-to-human transmission of COVID-19 has occurred among close contacts since the middle of December 2019 (Li et al., 2020). The new virus infection caused clusters of severe respiratory illness similar to severe acute respiratory syndrome coronavirus and was associated with ICU admission and high mortality (Huang et al., 2020)..

3. Methodology

The study was conducted at the Federal Polytechnic by evaluating the effectiveness of the automatic disinfectant booth mounted at the school gate for pedestrian use. The evaluation was done through the combination of survey research and application of queuing model.

Direct observation survey was conducted to determine the arrival rate and service rate. To get both the arrival rate and service rate, a five day sample with two measurements was taken in the morning and afternoon each day and mean score was calculated for both service and arrival rate. Subsequently the application of queuing model $\frac{\lambda}{\mu c}$ was adopted to reflect the interaction among service rate, arrival rate and number of service facilities, while system intensity rule $\frac{\lambda}{\mu c} < 1$ was applied to determine the effectiveness of the system.

A cursory look at the queuing system at the first gate of the Federal Polytechnic, revealed a multi-channel system which comprised of five (5) waiting lines (queue) to five service point/facilities (disinfecting booths). The expected waiting line length (queue length) was 20 students per each of the five (5) waiting lines and that made the total number of students expected to be on queue to be 100 while the total expected to be served was five and the total to be on the system per time therefore was one hundred and five (105). That means:

The queue length "L_q" = 20

Waiting line (Queue) = 5

And the service limits "C" = 5

Therefore, the total number of people expected in the system per time for it to be safe for the students and serve the purpose for which it was made for was i.e. the total number of students on each of the waiting lines which must not exceed 20 multiply by number of waiting lines (5) plus number of student being served in the facilities which could not exceed five (5) per time as well.

n = Total number of students in the system per time

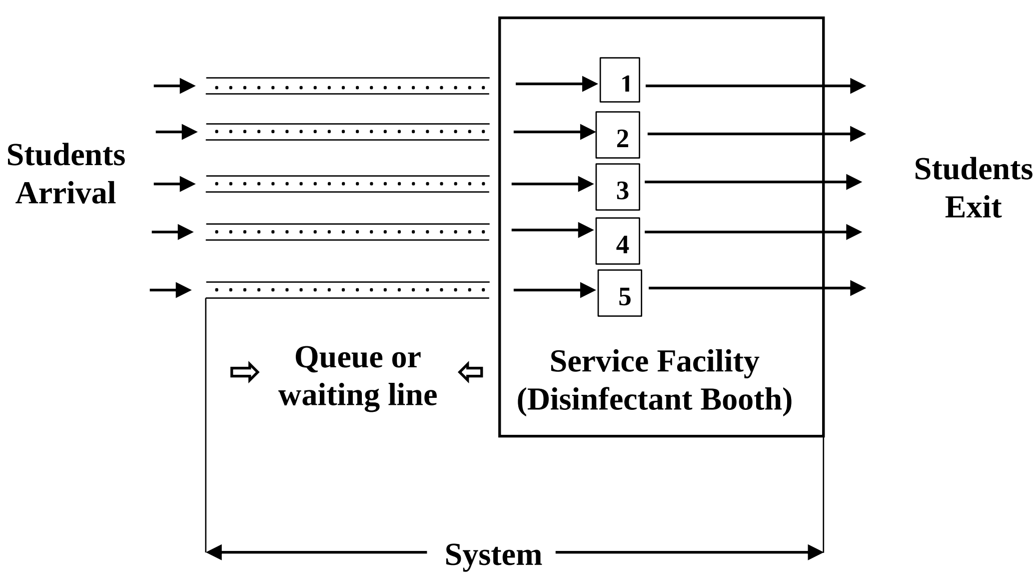
$n = Lq \times 5 + C$

$n = 20 \times 5 + 5 = 100 + 5 = 105$

:. Total number of students in the system per time = 105

The structure of the queue system was depicted in a diagram thus.

MULTI-CHANNEL QUEUING SYSTEM FOR DISINFECTANT SYSTEM AT THE FIRST GATE OF FEDERAL POLYTECHNIC, ILARO.



To conduct the survey six (6), enumerators were engaged and briefed on the procedure required. One of them took samples on the waiting lines for the purpose of computing the arrival rate, while the other five took samples on each of the disinfecting booths for the purpose of computing the service rate.

Two measurements were taken, each day for five consecutive days and mean scores '$\bar{X}$' were calculated for each of the days before the mean of the means $\bar{X}$ for previously calculated averages were calculated to get the value of both the service rate and arrival rate. Having known the value for the arrival rate, service rate and the number of service facilities, the system intensity was calculated and subsequently compared to the established threshold for decision taking.

The model for the study was presented as:

(a) Survey research

The arrival rate and service rate mean score

$$\bar{\bar{X}} = \frac{\sum_{i=1}^{n} \bar{X}_i}{n}$$

$$= \frac{\bar{X}_1 + \bar{X}_2 + \bar{X}_3 + \bar{X}_4 + \bar{X}_5}{n}$$

Where;

$\bar{\bar{X}}$ is the mean score for the five days

$\bar{X}$ is the mean of the two measurements taken per day.

(b) Queuing model for the system effectiveness system intensity = $\frac{\lambda}{\mu c} < 1$

Where;

λ = Arrival Rate

μ = Service rate

c = number of service point or system facilities

H0: There is no significant influence of the disinfecting booth in prevention of Covid-19

4. RESULT

4.1 Output of the survey conducted on arrival rate

Sample No	Measurement Per Sample		$X_1 + X_1$	$\frac{X_1 + X_1}{2} = \bar{X}$
	Morning (X_1)	Afternoon (X_2)		
1.	547	530	1077	$\bar{X}_1 = 539$
2.	544	530	1074	$\bar{X}_2 = 537$
3.	535	521	1056	$\bar{X}_3 = 528$
4.	539	529	1062	$\bar{X}_4 = 531$
5.	491	640	1136	$\bar{X}_5 = 568$

Computation of arrival rate (λ)

$$\bar{\bar{X}} = \frac{\sum_{i=1}^{n} \bar{X}i}{n}$$

$$\bar{\bar{X}} = \frac{\bar{X} + \bar{X} + \bar{X} + \bar{X} + \bar{X}}{5}$$

$$\frac{539 + 537 + 528 + 531 + 568}{5}$$

$$\lambda = \frac{2703}{5} = 541$$

The arrival rate (λ) = 541

4.2 Output of the survey conducted on the service rate

Sample No	Measurement Per Sample		$X_1 + X_1$	$\frac{X_1 + X_1}{2} = \bar{X}$
	Morning (X_1)	Afternoon (X_2)		
1.	172	192	364	$\bar{X}_1 = 182$
2.	177	183	360	$\bar{X}_2 = 180$
3.	186	181	366	$\bar{X}_3 = 183$

4.	180	184	364	$\bar{X}_4 = 182$
5.	176	185	361	$\bar{X}_5 = 181$

Computation of the service rate (μ)

$\bar{\bar{X}} = \frac{\sum_{i=1}^{n} \bar{X}i}{n}$

$\bar{\bar{X}} = \frac{\bar{X} + \bar{X} + \bar{X} + \bar{X} + \bar{X}}{5}$

$\frac{182 + 180 + 183 + 182 + 181}{5}$

Where;

$\bar{\bar{X}}$ is the mean score for the five days

$\bar{X}$ is the mean of the two measurements taken per day.

$\mu = \frac{908}{5} = 181.6$

$\underline{\mu} = 182$

4.3 Output of the queuing model (Traffic Intensity)

Trafic intensity

$= \frac{\lambda}{\mu C}$

$= \frac{541}{182 \times 5}$

$= \frac{541}{910}$

= 0.595

4.4 Decision Rule

$\frac{\lambda}{\mu c} < 1$

:. 0.595 < 1

The decision rule is that the system is explosive if the traffic intensity is more than 1 and considered effective when the traffic intensity is between 0 and 1. However, the closer to zero the better is the result.

5. Discussion

The value of the result gotten from the given model i.e 0.595 above indicates that the value was within the prescribed threshold of zero (0) to one (1) which implies that the system was effective and have influence on the prevention of Covid-19. But it should be noted that the influence was only moderate and not significant because the value at the middle point of the threshold of “0 to 1”. Since it is part of the decision rule that the closer the value is to zero “0” the better and significant it is. We can therefore infer from the foregoing that the system needs to be improved further.

It is also worthy of note that the staggered class attendance put in place has significantly controlled the arrival rate and prevented the system from being explosive.

6. Conclusion and Recommendation

Based on the findings of the study, it was concluded that the queuing system resulting from the disinfecting process at the Federal Polytechnic, Ilaro gate was effective and served the purpose of preventing the spread of covid-19 within the campus.

The staggered class attendance of students put in place by the school authority was able to control the “calling source” i.e student arrival at the gate, and in effect the population expected in school at a particular point in time.

The result gotten from the study indicates that despite being effective the disinfecting system needed to be improved. It is therefore recommended that;

- The Management of the institution should provide more disinfecting booths at the gate.
- The staggered class attendance arrangement should continue and monitored more effectively to ensure that all students not expected on campus at a particular time do not gain access into the campus.
- The Management should work on the disinfecting booths to enhance speed and efficiency of the service rate.

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